Synthesizing Conjunctive and Disjunctive Linear Invariants by K-means++ and SVM

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Abstract: The problem of synthesizing adequate inductive invariants lies at the heart of automated software verification. The state-of-the-art machine learning algorithms for synthesizing invariants have gradually shown its excellent performance. However, synthesizing disjunctive invariants is a difficult task. In this paper, we propose a method k++ Support Vector Machine (SVM) integrating k-means++ and SVM to synthesize conjunctive and disjunctive invariants. At first, given a program, we start with executing the program to collect program states. Next, k++SVM adopts k-means++ to cluster the positive samples and then applies SVM to distinguish each positive sample cluster from all negative samples to synthesize the candidate invariants. Finally, a set of theories founded on Hoare logic are adopted to check whether the candidate invariants are true invariants. If the candidate invariants fail the check, we should sample more states and repeat our algorithm. The experimental results show that k++SVM is compatible with the algorithms for Intersection Of Half-space (IOH) and more efficient than the tool of Interproc. Furthermore, it is shown that our method can synthesize conjunctive and disjunctive invariants automatically.

Keywords: Software verification, conjunctive invariant, disjunctive invariant, k-means++, SVM.

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1. Introduction

With the increasing size and complexity of software, it is more difficult and complex to verify the correctness of the software. Thus, how to ensure the correctness of software has aroused enough attentions [19, 26, 27]. In order to handle this problem, one of the popular techniques is the software verification [4].

However, the limitations of manual verification are becoming more and more obvious, and software verification technology needs innovative development. In recent years, the automated techniques and tools for software verification have gradually become an important research direction [10].

In the process of automated program verification, synthesizing inductive invariants plays a key role [28]. An invariant means that it is closed with respect to the transition relation of the program, and it guarantees that any execution of a statement in the program changes from a state that belongs to the invariant region to other state. Once adequate inductive invariants have been found, the problem of software verification can be reduced to logical validity of verification conditions, which are solved with the advances in automated logic solvers [7, 8]. What’s more, invariants also can be useful for compiler optimization, program understanding [18], bug detection [5].

In the past, people have put forward a lot of solutions to synthesize inductive invariants including model checking, abstract interpretation [3], Craig’s interpolation [20]. Although, these techniques are able to compute invariants, they have their own inherent hardness, accompanied by certain limitations. For example, model checking can successfully synthesize invariants when the program has a finite state-space or the paths in the program are bounded. However, for programs over an infinite domain, such as integers, with unbounded number of paths in the program, model checking is doomed to fail [7]. With the rapid development of artificial intelligence, the state-of-the-art machine learning algorithms, such as Support Vector Machine (SVM) [17, 24], decision trees [8, 15], and learning using Examples, Counter-examples, and Implications (ICE) [6], have been applied to synthesize invariants in recently years.

Recently, a great deal of approaches are based on guess-and-check model to synthesize invariants [6, 8 17, 22, 23, 24, 25]. Roughly speaking, those models regard the problem of synthesizing invariants as two parts, learner and teacher. The learner synthesizes candidate invariants and the teacher checks whether the candidate invariants are true invariants by a set of theories founded on Hoare logic in each round. If not, this model should give more details for learner to revise candidate invariants until the candidate invariants pass the checking. A common problem with the guess-and-check model is that their effectiveness is often limited by the samples collected in the first phase [17]. The paper [17] gives us an ideal that we can use selective samples to improve the efficiency.
In this paper, we develop a new method, called k++SVM, based on k-means++ [1], SVM [2], Hoare logic [13, 23, 25], selective samples and the guess-and-check model to synthesize invariants. At first, we collect samples and then cluster positive samples by k-means++. Secondly, SVM is adopted to separate positive samples from negative samples and get the hyper plane equations. The final step is to check the correctness of our results by Hoare logic.

1.1. Our Contributions
This paper makes the following contributions:

- We propose a method based on k-means++, SVM, Hoare logic, selective samples and the guess-and-check model to synthesize not only conjunctive invariants but disjunctive invariants as well.
- We show that our method has the ability to automatically synthesize conjunctive and disjunctive invariants. Actually, automatically synthesizing conjunctive and disjunctive invariants is a big challenge. Our method solves this problem in a unique way.
- We have implemented our method in Python for synthesizing invariants and we compare it with the algorithm Intersection of Half-space (IOH) and a tool named Interproc. The experimental results show that our method is compatible with IOH. Furthermore, it is shown that our method can synthesize conjunctive and disjunctive invariants automatically while the IOH and Interproc cannot.

1.2. Organization
The rest of the paper is organized as follows. We simply introduce our method by the way of two examples in section 2. Next, section 3 reviews related work. In section 4, we describe necessary material including sampling, k-means++, algorithms for intersection of half-space and Hoare logic. And then we give a detailed process of k++SVM in section 5. In section 6, we show the experimental results and verify the effectiveness of our algorithm. Finally, section 7 concludes with some directions for future work.

2. The Motivating Examples
2.1. Conjunctive Invariants
Considering the program in Figure 1, it has two integer variables x and y. After passing two loops, the value of x and y have changed. In the end, we should check whether the bad state error() can be reached by the value of y. If the error() state cannot be reached, the initial values of x, y and their metabolic values in the program paths are good states. If we give a arbitrary values of x, y, then the error() state is reached after executing the program. Only the initial values of x, y, except their values in the path, are bad states. Suppose we consider a path that goes through the two loops once. We sample these two points \{(0, 0), (1, 1)\} as good states and points \{(0, 1), (1, 0)\} as bad states. Figure 1 gives the codes and plots the distribution of these four points. The solid points represent positive samples and the hollow points represent negative samples.

By observing the program, we find the tendency of the variables is linear, thus only linear inequalities, rather nonlinear inequalities, can better characterize the intrinsic properties of this program. So we adopt the machine learning algorithm of linear kernel SVM to obtain the linear inequalities. Unfortunately, the distribution of this four points cannot be separated by using linear kernel SVM once. However, there exist two linear inequality \(2y < 2x + 1\) and \(2y > 2x - 1\) which are able to represent invariants (see the two straight lines in Figure 1). Our solution is that, firstly, we use k-means++ to cluster the positive samples \{(0,0), (1,1)\} as one cluster in the condition the value of k is 1. If we choose the negative sample \{(0, 1)\} with positive sample cluster and use linear kernel SVM, we can get the invariant \(2y < 2x + 1\). Similarly, if we choose another negative samples \{(1, 0)\} with positive sample cluster and use SVM, we can get the invariants \(2y > 2x - 1\). After we take the intersection of two inequalities, we can get the result \(2y > 2x - 1 \land 2y < 2x + 1\). Because the type of x and y is integer, our result can be equivalent to \(y \geq x \land y \leq x\). And then we can regard x=y as final result. Finally, by a set of theories of Hoare logic, it can be checked that x=y is an invariant which is sufficient to prove the error() state is unreachable.

2.2. Disjunctive Invariants
Considering the program in Figure 2, we sample in the same way as example 1. For instance, if we assume the values of x and y are \{(0, -3)\}, then the states we reach are \{(-1, -2), (-2, -1), (-3, 0)\} and thus these are all good states. Similarly, if we define their values as \{(-2, 2)\}, then the states we reach are \{(-1, 1), (0, 0)\} which violate the assertion and thus \{(-2, 2)\} is a bad state. And if we directly define their values are \{(0, 0)\}, then the loop will not run and the value of x is 0 which violates the assertion as well. We plot samples in a coordinate system.
In these circumstances, positive constraints for leaner to refine invariants. This synthesis of good states and bad states. Suppose that we a transition sequence from an initial state to a bad state. Reach is the set of reachable states and I is an adequate inductive invariant that to distinguish these two states [7] (see Figure 3). If we can assure any state in the program belongs to Init and Reach, this program is correct.

The above interpretations guide us how to sample. A good state is defined as any state that the program and can satisfy a specified safety specification while bad states cannot satisfy. If the program is correct, then there is no transition sequence from an initial state to a bad state. Reach is the set of reachable states and I is an adequate inductive invariant that to distinguish these two states [7] (see Figure 3). If we can assure any state in the program belongs to Init and Reach, this program is correct.

The method was a typical form of guess-and-check model. The paper [23] described a general framework for computing invariants by iteratively executing two phases. The search phase applied randomized search to discover candidate invariants and the validate phase applied checker to validate the correctness of the candidate invariants. The paper [22] proposed a data driven approach for generating algebraic polynomial loop invariants. Firstly, the method collected the value of the variables by executing a specific program. Secondly, it obtained a data matrix according to a template and upper bound d. Next, this method used linear algebra techniques to compute a candidate invariant. Finally, it checked the effectiveness of the candidate invariants. Our work is inspired by above paper. Our method uses machine algorithms to synthesize invariants, adopts guess-and-check model to ensure the correctness of the invariants and collects selective samples to reduce the number of iterations.

4. Preliminaries

4.1. Sampling

If we regard a program as a transition system, the states of program can be split in two states—good states and bad states—when we execute the program. The good states are states that include the initial states of the program and can satisfy a specified safety specification while bad states cannot satisfy. If the program is correct, then there is no transition sequence from an initial state to a bad state. Reach is the set of reachable states and I is an adequate inductive invariant that to distinguish these two states [7] (see Figure 3). If we can assure any state in the program belongs to Init and Reach, this program is correct.

The above interpretations guide us how to sample. A good state is defined as any state that the program could conceivably reach when it is started from a state consistent with the precondition [20]. In other words, the initial value and their changed values which can make bad state is unreachable are both positive samples. Similar, bad states are defined as states, except the states in program path, that will make assertion fail after executing the program. Thus, the values that will make assertion fail represent the negative samples.

In the following, we give the formalized description of good states and bad states. Suppose that we are given a Hoare triple in the following form:
We assume that s represents those state before Pre, s’ are the states in the loop body, and s’’ are the states out of loop body but before the Post. The positive samples a negative samples are:

Positive states = 
\[
\begin{align*}
&\text{s} \in \text{states}, \text{s’} \in \text{states}, \text{s’’} \in \text{states} \\
&\text{s} \in \text{Pre} \land \text{s} \rightarrow \text{s’} \rightarrow \text{s’’} \\
&\land \text{s’’} \notin \text{Post}
\end{align*}
\]

Negative states = 
\[
\begin{align*}
&\text{s} \in \text{states}, \text{s’} \notin \text{states}, \text{s’’} \notin \text{states} \\
&\text{s} \notin \text{Pre} \land \text{s} \rightarrow \text{s’} \rightarrow \text{s’’} \\
&\land \text{s’’} \notin \text{Post}
\end{align*}
\]

Algorithm 1: IOH

\[
\begin{align*}
\text{Input:} & \quad \text{Positive sample: X+} \\
& \text{Negative samples: X-} \\
& \text{Cost parameter: } c := 1000
\end{align*}
\]

\[
\begin{align*}
\text{Output:} & \quad \text{Candidate Invariants::} H \\
1: & \quad H := \text{true} \\
2: & \quad M := X- \\
3: & \quad \text{While } |M| \neq 0 : \\
4: & \quad \text{Arbitrarily choose } b \text{ from } M \\
5: & \quad h := \text{Process (SVM (X+, \{b\}), X+, X-)} \\
6: & \quad \forall b’ \in M \text{ s.t. } h(b’) < 0 \\
7: & \quad \text{Remove } b’ \text{ from } M \\
8: & \quad H := HAh \\
9: & \quad \text{End While} \\
10: & \quad \text{Return } H
\end{align*}
\]

We realize IOH by the computer language of Python and apply it for the samples generated by make_circles function in the Python packages of sklearn. The result is showed in Figure 4. The circle points and triangle points represent negative samples and positive samples respectively. We can get the conclusion that IOH is feasible and inspiring.

\[
\begin{align*}
\text{Figure 4. A inspiring result for IOH.}
\end{align*}
\]

4.4. Hoare Logic

Hoare logic [22], also known as Floyd-Hoare logic, is a formal system developed by British computer scientist Hoare [13]. The purpose of this system is to provide a set of logical rules for the correctness of computer programs using strict mathematical logic.
reasoning. The central feature of the Hoare logic is the Hoare triple:

\[ \{P\} C \{Q\} \] (1)

In Hoare triple, \(P\) represents precondition, \(Q\) is the post-condition and \(C\) means the code segment. If a program is correct, it means that when \(P\) holds, after the execution of \(C\), \(Q\) holds upon exit.

As the program with loop, the invariant is loop invariant. Loop invariant synthesis is a huge challenge [16]. It is an assertion that has been held since the beginning of the execution of the loop until the end of the loop. Just let a theorem prover execute only once, we can check whether invariant \(I\) is true. Here is the formal description of the program with loops:

\[ \{P\} \text{while } B \{C\} \{Q\} \] (2)

\(B\) represents loop condition and \(C\) is the code in loop. According to Hoare logic, given an invariant \(I\), by decomposing the formal description, we can get these two results:

\[ \{I \land B\} C \{I\} \] (3)

\[ \{I \land \neg B\} \{Q\} \] (4)

On the basis of existing theories, we can confirm that \(I\) is a correct invariant as long as \(\{P\} \Rightarrow \{I\}\), \(\{I \land B\} C \{I\}\) and \(\{I \land \neg B\} \Rightarrow \{Q\}\). The sign of \(\Rightarrow\) means implicating. For example, \(x=y\) implicates \(x\geq y\). In other words, while the condition \(x=y\) holds, \(x\geq y\) also holds.

In our experiments, the \(I\) in the program of Figure 1 is \(x=y\), and \(P\) is \(x=y=0\). \(\{P\} \Rightarrow \{I\}\) without any doubt. \(\{I \land B\} C \{I\}\) holds, because the changes of \(x\) and \(y\) are the same in \(C\). \(B\) is \(x! = 0\), so \(\neg B\) is \(x=0\). \(I \land \neg B\) means \(x=y\) and \(x=0\). We can get a conclusion that \(y=0\) which can avoid program execution, and the program statement of error() is a true invariant.

If we found that \(I\) is an error invariant by theorem prover, we need to sample more. Specifically, if we just sampled three points \{(0, 1), (0, 0), (1, 1)\}, we can get an invariant \(y < x - 0.5\) by using our algorithms. Because the type of variables \(x\) and \(y\) are integer, the invariant is equal to \(x \geq y\). \(\{P\} \Rightarrow \{I\}\) holds, but \(\{I \land \neg B\} \Rightarrow \{Q\}\) does not hold. It means some bad states are contained by \(I\), so we should collect more samples and repeat our algorithm.

The above is a theoretical proof. In the actual experiment, Z3 theorem prover [21] helps us to verify the correctness of candidate invariants.

5. K++SVM

IOH is inspiring, but there are still problems waiting to be solved. Considering the program in Figure 2, after sampling, it is found that the positive samples are distributed among multiple clusters. IOH just choose one negative sample, it is surprised to discover IOH is infeasible under this situation. Thus, k++SVM emerges as the times required for solving this kind of problem. At first, we follow a program to sample positive samples and negative samples. Then, k++SVM defines the value of \(k\) as 1 at the beginning, and then adopts k-means++ algorithm to cluster the positive samples as \(k\) clusters. Next, our algorithm uses linear kernel SVM to separate samples. If samples can be separated, we get the candidate invariants. Otherwise, SVM is used to eliminate single negative samples one by one for getting the expression of hyperplane equations. If it is valid, the candidate invariants are found. If not, this algorithm lets the value of \(k\) add 1 and cluster the positive samples again. Then repeat algorithm till it can distinguish samples and get the candidate invariants. Finally, we should check whether the candidate invariants are true invariants. If the check succeeds, we get the final result. Otherwise, we should sample more states and repeat our algorithm.

Our algorithm is based on the guess-and-check model, some common problems with this model are that the effectiveness of algorithm is limited by the samples not only generated in the first phases [17], but also generated when the check fails. If the distribution of sampling points is reasonable, the iteration numbers between guess and check are fewer and the effectiveness of algorithm is higher. The most ideal situation is that our algorithm get the true invariants just only using guess-and-check model once. This situation is hard to achieve, but we have solutions to get close to this situation. Firstly, a large number of samples are needed, which can make sure that they include the samples characterizing the program. In fact, only the samples which can characterize the program are participated in the construction of invariants, because the support vector is only related to those samples. Secondly, we get the heuristics from paper [17]. According to this paper, sampling through verification provides useful new samples by paying a high cost. So it applies method selective Sampling() to selectively generate new sample while are closed to the candidate invariants. Our method also collect selective samples to improve the efficiency of algorithm.

The condition for the change of the value of \(k\) is that IOH cannot get the candidate invariants, but this description is not an algorithmic language. In fact, in the process of algorithm implementation, we need to change the value of \(k\) as long as the capacity of \(M\) is not reduced. Every time, we choose one negative sample and then use SVM to separate the one negative sample from positive samples (or positive samples cluster). That is to say, every time we can at least separate out one negative sample. If the capacity of \(M\) reduces, we continue to execute the algorithm. Otherwise, we stop IOH and let the value of \(k\) add 1. Here we give the pseudo code (see Algorithm 2).
Algorithm 2: k++SVM

Input: Positive sample: X+
       Negative sample: X-
       Cost parameter: c := 1000

Output: True Invariants: H

1: k := 1
2: Cluster(s+x+k), obtain X: {X1+, X2+,..., Xk+}
3: If (SVM can be used): 
   4: For i=1 to k (i=k is allowed): 
      5: h := Process (SVM (Xi+, {b}))
      6: H := HV/h
5: End For
6: End If
7: End While
8: Else:
9: For i=1 to k (i=k is allowed): 
   10: M := X-
   11: While | M | ≠ 0 :
      12: Arbitrarily choose b from M
      13: h := Process (SVM (Xi+, {b}))
      14: ∨ h' = M s.t. h (h')<0 
      15: Remove b' from M
      16: If (| M | || | M |) :
          17: k := k+1
          18: Goto 2 
      19: Else:
          20: Hi := h ^ hi 
      21: End While
      22: H := HV/H
23: End For
24: add selective samples
25: Goto (2 only once)
26: If (H is indeed invariants): 
   27: Return H
28: Else:
29: Add more samples to X+ and X-
30: Goto 2 

6. Implementation and Evaluation

We have implemented a prototype version of the algorithm described in this paper in the computer language of Python. First, we obtain constraint expression file whose type is txt by using CBMC tool (http://www.cprover.org/cbmc/). CBMC is a Bounded Model Checker for C and C++ programs and, just right, all of our experimental programs are C programs. Secondly, according to the constraint expression file, we generate samples by using the z3 Satisfiability Modulo Theories (SMT) solver [21] for constraint solving. Next, to realize our algorithm and show result, numpy, sklearn, codecs, re and matplotlib packages of Python are adopted. Finally, we use SMT solver [21] to check whether the candidate invariants are true invariants.

The k-means++ and SVM in k++SVM are supported by off-the-shelf sklearn packages of Python. There is an important problem concerning the value that the cost parameter c of SVM algorithm should take. In order to guarantee that under certain conditions the programs will never violate assertion, our classifier is not allowed to misclassify. A low value of c allows the generated classifier to make errors on the training data. So, we assign a very high value to c (1000 in our experiments). We conduct all of the following experiments on a core i5 CPU with 8GB of RAM running Windows 10.

The results are shown in Tables 1 and 2. In Table 1, Num is the experiment number. File is the name of the program, and we can find it in the reference behind the name; LOC are code lines; Invariants are the invariants of the program output by using IOH. Thereinto, Fail means that finding invariants by IOH is failed; Our Invariants are the invariants output by using k++SVM; Total means the total number of the samples including positive samples and negative examples; K is the value of k in k++SVM; we can find the code of 13th experiment in Figure 5.

![Figure 5. The code of 13th.](image-url)

In Table 2, Times and our times is running time of the IOH and k++SVM respectively. The statistical time is from input samples to output candidate invariants.

<table>
<thead>
<tr>
<th>NUM</th>
<th>File</th>
<th>LOC</th>
<th>Invariants</th>
<th>Our Invariants</th>
<th>Total</th>
<th>K</th>
<th>Type</th>
<th>Interproc</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Figure 7 [10]</td>
<td>8</td>
<td>x&lt;=8</td>
<td>x&lt;=8</td>
<td>22</td>
<td>1</td>
<td>con</td>
<td>Y</td>
</tr>
<tr>
<td>2</td>
<td>Figure 1-a [24]</td>
<td>14</td>
<td>(y &gt; x - 0.5) &amp; (y &lt; x + 0.5)</td>
<td>(y &gt; x - 0.5) &amp; (y &lt; x + 0.5)</td>
<td>40</td>
<td>1</td>
<td>con</td>
<td>Y</td>
</tr>
<tr>
<td>3</td>
<td>ex1 [13]</td>
<td>22</td>
<td>w + 2w &gt; 2</td>
<td>w + 2w &gt; 2</td>
<td>36</td>
<td>1</td>
<td>con</td>
<td>N</td>
</tr>
<tr>
<td>4</td>
<td>Figure 2 [9]</td>
<td>18</td>
<td>3x &gt;= y</td>
<td>3x &gt;= y</td>
<td>62</td>
<td>1</td>
<td>con</td>
<td>Y</td>
</tr>
<tr>
<td>5</td>
<td>fse06 [10]</td>
<td>8</td>
<td>((y &gt; -0.5) &amp; (x &gt; -0.5))</td>
<td>(y &gt; -0.5) &amp; (x &gt; -0.5)</td>
<td>36</td>
<td>1</td>
<td>con</td>
<td>N</td>
</tr>
<tr>
<td>6</td>
<td>Figure 1[12]</td>
<td>6</td>
<td>x - n &lt;= 0</td>
<td>x - n &lt;= 0</td>
<td>43</td>
<td>1</td>
<td>con</td>
<td>Y</td>
</tr>
<tr>
<td>7</td>
<td>Figure 1-a [17]</td>
<td>8</td>
<td>x &lt;= y + 16</td>
<td>x &lt;= y + 16</td>
<td>100</td>
<td>1</td>
<td>con</td>
<td>Y</td>
</tr>
<tr>
<td>8</td>
<td>Figure 1-b [17]</td>
<td>9</td>
<td>Fail</td>
<td>x &gt; 0 &amp; y &gt; 0</td>
<td>82</td>
<td>2</td>
<td>dis</td>
<td>N</td>
</tr>
<tr>
<td>9</td>
<td>Figure 1-d [17]</td>
<td>6</td>
<td>Fail</td>
<td>x &lt; 0 &amp; y &gt; 0</td>
<td>120</td>
<td>2</td>
<td>dis</td>
<td>N</td>
</tr>
<tr>
<td>10</td>
<td>Figure 1 [15]</td>
<td>10</td>
<td>Fail</td>
<td>(y - x + 0.5) &amp; (y &lt; x - 0.5)</td>
<td>50</td>
<td>2</td>
<td>dis</td>
<td>N</td>
</tr>
<tr>
<td>11</td>
<td>Figure 4 [11]</td>
<td>8</td>
<td>Fail</td>
<td>(x &lt; 0.5) &amp; (y &lt; 0.5)</td>
<td>40</td>
<td>2</td>
<td>dis</td>
<td>N</td>
</tr>
<tr>
<td>12</td>
<td>Figure 1 [11]</td>
<td>8</td>
<td>Fail</td>
<td>(y &lt; 0.5) &amp; (x &lt; 0.5)</td>
<td>42</td>
<td>2</td>
<td>dis</td>
<td>N</td>
</tr>
<tr>
<td>13</td>
<td>Quad</td>
<td>6</td>
<td>Fail</td>
<td>(x&lt;0.5 &amp; y&gt;0.5) &amp; (x&lt;0.5 &amp; y&lt;0.5)</td>
<td>46</td>
<td>2</td>
<td>dis</td>
<td>N</td>
</tr>
</tbody>
</table>
In the last column of Table 1, Interproc means whether the true invariants can be synthesized by the verifier Interproc. Y and N respectively indicate that it can or cannot synthesize true invariants. Interproc [14] is an interprocedural analyzer for a small imperative language with procedure calls. In the basis of the technique of abstract interpretation, it infers invariants on the numerical variables of analyzed program.

The invariant comparison of experimental results are shown in Table 3. Num is the experiment number in Table 1; Reference means the source of the code; Invariants represent the results given in source; Our Invariants are the results obtained by our method.

<table>
<thead>
<tr>
<th>NUM</th>
<th>Reference</th>
<th>Invariants</th>
<th>Our Invariants</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>[24]</td>
<td>x = y</td>
<td>(y &gt; x - 0.5) ∨ (y &lt; x + 0.5)</td>
</tr>
<tr>
<td>5</td>
<td>[28]</td>
<td>(y &gt;= 0) ∨ (x &lt;= 0)</td>
<td>(y &gt; -0.5) ∨ (x &lt; 0)</td>
</tr>
<tr>
<td>10</td>
<td>[15]</td>
<td>x + y &gt; 0</td>
<td>(y &gt; -x + 0.5) ∨ (y &lt; x - 0.5)</td>
</tr>
<tr>
<td>12</td>
<td>[11]</td>
<td>(x &lt;= -1) ∨ (y &gt;= 1)</td>
<td>(x &lt; -0.5) ∨ (y &gt; 0.5)</td>
</tr>
</tbody>
</table>

Comparing our experimental results with other paper, there is nothing different between them (see Table 3). For example, in our 12th experiment, the invariants are (x < -0.5) ∨ (y > 0.5), but the result in paper [11] is (x <= -1) ∨ (y >= 1). In our 5th experiment, the invariants are (y > -0.5) ∨ (x > -0.5), but in paper [4] the invariants are (y >= 0) ∧ (x >= 0). The reason is that the type of the variables is integer, and the hyperplane parameter exported by SVM are the type of float. Just change our results to 0.5 units, we can get the same results. More specifically, in our 2nd experiment, by simplifying our result, we can get the conclusion (y >= x) ∧ (y <= x). As the final result, our invariant is x = y which is the same as the result in paper [24].

![Figure 6](image.png)

In section 3, 4, we prove that x=y is the true invariant. Suppose we use other machine learning algorithms, rather than linear kernel SVM, we get a nonlinear invariant (the form of circle in Figure 6). We also can prove that this kind of result is wrong. This gives us a heuristic idea. For a specific program, we are not supposed to only pursue that the algorithms can distinguish positive and negative samples, in spite of the essential characteristics of the change of the program variables. This point is well verified why the form of invariants should be linear.

In order to evaluate our method in synthesizing invariants, we investigate the following research question:

- **RQ1** the first research question which we would like to answer to is: does our method help to synthesize disjunctive invariants?

Through the 8–13th experimental results in Table 1, it is obvious that the invariants with the form of disjunctive can only be computed by our method, while the IOH and Interproc cannot. Compared with Interproc, it is because that our method regards the problem of synthesizing invariants as the problem of classification by sampling and using classification algorithm. By analyzing samples rather than specific procedural logic, our method can synthesize disjunctive invariants. Not like the IOH, our method should cluster the positive samples before classification. According to our results, whether the target invariants are conjunctive or disjunctive depend on the number of the positive sample’ clusters. If the number of positive sample’ cluster, that is the value of k, is one, the target invariants are conjunctive. Otherwise, they are disjunctive. However, the number of the positive sample’ clusters are not determined by the distance, but, intrinsically, the situation that whether the positive samples clusters can be separated clearly from negative samples. This point is also just reflected in the condition that the value of k should be changed when the samples cannot be separated clearly.

- **RQ2** the second research question which we would like to answer is: does our method have the ability to automatically synthesize invariants? IOH and Interproc cannot synthesize disjunctive invariants and Interproc cannot compute the invariants in 3rd and 5th experiment. What’s more, when we using Interproc, we should manually choose the form of target invariant first. Specifically, only when we choose convex polyhedra option to complete our 4th experiment and choose octagon option to complete 2nd experiment, Interproc is able to get the right invariants. This is a contradictory problem. In fact, before executing Interproc, we don’t know which option should be chosen to get the right results. Through our experimental results, only our method can automatically synthesize disjunctive invariants.
Because our method obtains the program feature by the relation of samples, rather than the logic relation in a program. And then our method judges that the invariants are conjunctive or disjunctive by the value of \( k \). If \( k \) is equal to 1, the invariants are conjunctive. Otherwise, they are disjunctive. The way to determine the value of \( k \) is that we gradually increase the value of \( k \) by 1 with the condition the positive samples cannot be separated clearly from negative samples. The optimal value of the \( k \) can be founded by the way of iterating. That is just the reason why our method have ability to automatically synthesize invariants.

- **RQ3** the third research question is: how about the performance between IOH and our method? As the disjunctive invariant, IOH is useless. As the conjunctive invariant, referring to the Table 2, the running time of k++SVM is closed to IOH. Generally speaking, the running time of k++SVM is slightly higher than IOH, but sometimes slightly lower. It is reasonable that k++SVM is slightly higher, because k++SVM is more complex and should use k-means++ algorithm extra. The situation that k++SVM is slightly lower is unexpected, but it is still reasonable, because both the k++SVM and IOH are defective. When they use SVM, they choose one negative sample randomly. This randomness results that the iterations of SVM cannot be controlled, although it does not have a great impact on the results. This is the point that we will research in the future.

- **RQ4** the fourth research question is: how about the performance in synthesizing disjunctive invariants by our method? Generally, referring to the Table 2, the running time of synthesizing disjunctive invariants is much higher than the running time of synthesizing conjunctive invariants by our method. It is reasonable, because the method of synthesizing disjunctive invariants have repeatedly used the method of synthesizing conjunctive invariants. In any case, because the time unit is still in the millisecond level, the running time of synthesizing disjunctive invariants by our method is acceptable.

### 7. Conclusions

We have shown that synthesizing invariants by machine learning algorithm can be profitable. In particular, when the target invariants are disjunctive, the k++SVM based on k-means++ and SVM is valid. In this paper, we use the theory method based on Hoare logic to prove the correctness of the invariants. Through our experiments, we have also verified that k++SVM is not only compatible with IOH, but can synthesize disjunctive invariants that IOH and Interproc cannot solve.

As the future work, we would like to modify our algorithm to eliminate the randomness and to make it more stable and robust.

### References


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