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Effects of Network Structures and Fermi Function's Parameter β in Promoting Information Spreading on Dynamic Social Networks

Abdulla Ally and Ning Zhang

Business School, University of Shanghai for Science and Technology, China

Abstract: Network represents a multitude of interactions through which information spreads within a society. Indeed, people are connected according to the way they interact with one another and the resulting network significantly determines the efficiency and speed of information spreading. This paper aimed at examining how topological structures of dynamic social network and Fermi function's parameter β influence information spreading. In order to carry out this study precisely, two models were proposed to generate a variety of network structures. To study the spreading process, the models were integrated with an epidemic Susceptible-Infected-Recovered (SIR) model and designed in such a way that nodes rewire network edges according to Fermi function which depends on a parameter β . By studying the number of recovered nodes generated in the spreading process and the number of acquainted nodes that are receiving information in each time step, the results suggested that network structure and both positive and negative β play an important role in promoting information spreading. These results give a good indication that the structure of a society influences the spreading process. More specifically, the structure of dynamic interactions is a good promoter of information spreading. Moreover, it is proposed that rewiring more than three edges of random network could yield no significant advantages in promoting information spreading. The present study likely enriches our knowledge and provides more insight on information spreading.

Keywords: Dynamic social network, information spreading, network structure, rewiring.

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1. Introduction

Information spreading plays a key role in changing and dealing with a society, by means of knowledge sharing [8], broad casting technological innovations [12], and so on, through their social connection [17]. The network structure of who is connected to whom becomes an important tool of information spreading [26, 29]. In real networks, frequent social activities of individuals shape the network evolution such as rewiring of new edges or removal of edges or nodes [11]. This network dynamic enhances information spreading process [10, 14]. However, the role it plays has to be considered in conjunction with other influences such as Fermi function's parameter β [14, 27] and social reinforcement [15]. Authors who previously devoted their interest to understanding information spreading mainly assumed a static society [14], a notable exception being that of Liu and Zhang [14].

Liu and Zhang [14] found that information spreading is very narrow for negative β while effective when selecting positive β . It is not clear that this result is general and will hold when the network structure is altered. However, their study mainly concerned rewiring the link to only second order friends; specifically, it did not consider the neighbours with no connections to first order friends. In real social network, interactions are far from being static: nowadays people are really dynamic and continuously make friends. It is believed that people can make friends without connection from the first order friends. Through interactions in the sport's playgrounds, people can make friends. This deliberation motivates us to raise a question about the network structures and the dependence of parameter β in promoting information spreading. In this paper, the authors aimed at studying the effects of topological structures of dynamic networks and Fermi function's parameter β in enhancing the speed and efficiency of information spreading. In order to carry out this study, two models were proposed to generate a variety of network structures. To study the spreading process, the present models were coupled with Susceptible-Infected-Recovered (SIR) model [2, 3, 9, 24] on four prototype complex networks: The Regular network (RG), the Erdős-Rényi (ER), the Watts-Strogatz (WS) and the Barabási-Albert (BA) models. The first model is a prototype example of RG with finite connectivity fluctuations [1]. However, the regular connectivity might be destroyed after rewiring with the present models; the second model is a random network with random connectivity fluctuations [1]. The third one is a small world network with bounded connectivity fluctuations. It interpolates between RG and random

network without changing the number of nodes or edges [1]. However, most of the nodes in this network are not neighbours of one another but can be reached from each other [13]. The last model is a prototype example of scale free network [1].

For proper comparative and comprehensive analysis, the same approach of study in [14] was maintained, where the network edges were rewired according to Fermi function from statistical mechanics. This function has been commonly used in the previous studies in [5, 7, 20, 21, 22, 28]. Within this frame, a number of different facets of analysis have been proposed in [14]. In this work, the focus was on investigating the number of recovered nodes and acquainted nodes that are generated as a result of the spreading process [6]. In view of this situation, it is appealing to compare the results demonstrated in [14] and discuss the spreading influence based on the structures proposed network in their work. Consequently, three models were considered in the present work: Model A, model B and the model in [14]. Analysis on a variety of structures generated by these models revealed that topology of a network, as shown below, has a great influence in information spreading. However, the speed and efficiency of spreading is also determined by the numerical value of parameter β .

2. Models Descriptions

The methodology implemented here follows that of Liu and Zhang [14] but strategies of rewiring network edges were different in the present models. We introduced two rewiring models to generate various structures based on the four prototype complex networks. In order to study the spreading process, the present models were integrated with an epidemiological model, the SIR model, which classifies the population into three categories according to their states [2]. Let's consider a population of N individuals that are represented as nodes in the network, where each node must occupy one of the three states: Susceptible node (S state)-will not inform others but may be informed; Acquainted node (I state)-have information and can transmit to susceptible nodes; and Recovered node (R state)recovered and thus will not take part in the spreading process. The link between them is a connection along which information can spread. In all the experiments, the spreading rate was set to 0.2 and the probability of recovery was set to 1. The study assumed that a seed node breaks old edge and rewire new edges with a probability $p = \frac{1}{1 + exp^{-\beta(\Pi_2 - \Pi_1)}}$, which depends on the payoff difference (n, -n) between two nodes. The

notation Π_1 and Π_2 represent the number of susceptible neighbours of the two targeting nodes: Anode with old edge connecting to the seed of information and a node to establish new connection, respectively. The parameter $\beta(-\infty \le \beta \le \infty)$ is considered as the ratio of Boltzmann distribution (Boltzmann's constant and temperature) for the two states of energy levels in statistical mechanics. Here, such parameter denotes the strength of selecting neighbours. Small β means the selection is nearly neutral whereas for large β selection turns out to be arbitrarily strong. Each model is described separately.

2.1. Model A

In the model A, an acquainted node breaks old network edge and rewires the edge to two different randomly selected susceptible nodes: One among the second order friends and other chosen from the whole network, following the above mentioned rescaled Fermi function [5, 7, 20, 21, 22, 28].

2.2. Model B

In the Model B, an acquainted node breaks old network edge and rewires the edge to three different randomly selected susceptible nodes: One among the second order friends and other two randomly selected from the whole network, following the above mentioned rescaled Fermi function [5, 7, 20, 21, 22, 28].

All the models assumed that nodes break old edge and refurbish their edges to friends with larger payoff (a node with large number of friends) [18, 23].However, it can also be possible to make friends to nodes with smaller payoff [7, 14]. In social networks, individual's degrees vary greatly, and highly connected individuals can spread information to a large number of peers if informed. The study assumed the same scenario as the common approach in which an individual interacts with the whole population and able to derive an individual payoff. This is similar to Tuyls and Parsons [23] noted in Rosenchein and Zlotkin [19].

In all the experiments, it was designed in such a way that at the beginning of the experiment, one acquainted node (I state) is randomly chosen from the whole network which, in this paper, considered as the source of information; and all other nodes were considered as susceptible (S state). The study assumed that at each time step, the acquainted node conveys information to susceptible nodes and then recovered (change to R state). The simulation continued until all acquainted nodes completely changed to R state.

2.3. Network and Experimental Settings

A series of experiments for the present models were performed on four different prototype complex networks: The RG, ER, WS and BA models. We simulated population structures that use network size N=10,000 and average degree $\langle k \rangle = 6$. In RG network, we designed a circular lattice (no randomness) with N nodes and *k* edges per node [1, 25], WS network: Each edge was rewired based on RG with a probability of 0.4 [1, 25]; ER network: All edges were randomly rewired with a probability of 1 and BA network was generated by the BA model with the number of edges for the new node set to 3 (k/2) [4, 14]. Each experiment was performed by setting various techniques namely static network, $\beta=2$, $\beta=0$ and $\beta=-2$. All the simulations run over 10,000 independent realizations for a given value of parameter β and static network technique.

3. Simulation Results and Discussion

3.1. Results of Recovered Nodes (*pr*) **Obtained** from Various Network Structures

We present results of simulations that were performed on various network structures generated by different models. The evolution of recovered nodes (pr) was examined. In general, a larger pr value indicates faster and broader information spreading [29]. Precisely, information spreads more effectively on a network structure which promotes high values of pr.

3.1.1. Results of The Model A

Figures 1, 2, 3, and 4 show the simulation results obtained when rewiring the networks with model A. The results revealed that for the same value of parameter β (say $\beta=2$), different network structures produce different values of pr to different extents. As a specific example, the structures obtained from ER, WS, RG and BA networks were found to produce 80.5%, 72.1%, 36% and 29.8% of pr, respectively when the value of β was set to 2. The structures of these networks obtained by static network technique produced 80%, 63.6%, 0.05% and 18.1% of pr, respectively. Analyzing the results closely, the network structure seems to hold a role in promoting the value of pr. On the other hand, for one particular network structure, a variety of pr ranges appeared in a structure are attributed by different values of β . For example, the values of pr obtained on structures generated from RG network were 36%, 10.4% and 0.04% when considering β =2,0 and -2, respectively as shown in Figure 1. A similar case was observed on the other networks. For instance, when the value of β was set to 2, 0 and -2 on BA network, the pr was enhanced to 29.8%, 26.5% and 28.8%, respectively as shown in Figure 4. To this end we are motivated that efficient spreading of information in a network can be affiliated to both the value of parameter β [7, 14] and the structure of a network [29] imposed. To test this hypothesis further, another set of simulations with similar set up as before were performed, but this time different network structures generated by the model B were considered.



Figure 1. Evolution of pr value on network structure generated after rewiring RG network with model A.



Figure 2. Evolution of pr value on network structure obtained after rewiring ER network with model A.



Figure 3. Evolution of pr value on network structure obtained after rewiring WS network with model A.



Figure 4. Evolution of pr value on network structure obtained after rewiring BA network with model A.

3.1.2. Results of the Model B

The simulations, as shown in Figures 5, 6, 7, and 8, revealed a similar case of results for much wider ranges of pr. Different network structures were found to generate different values of pr to different magnitudes when the same value of β was set in the experiments. The values of pr on structures obtained after rewiring RG, ER, WS and BA networks were different for $\beta=2$. The pr on these networks were promoted at magnitudes of 58.3%, 85.1%, 79.4% and 38.1%, respectively. Furthermore, the network structures generated by static network technique produced pr to different ranges. For example, pr on structure from RG network was promoted to only 0.05% while it was 80% on ER network as shown in Figures 5 and 6. This indicates that both network structure and positive β have a certain influence on the value of *pr*.



Figure 5. Evolution of pr value on network structure generated after rewiring RG network with model B.



Figure 6. Evolution of pr value on network structure generated after rewiring ER network with model B.



Figure 7. Evolution of pr value on network structure generated after rewiring ws network with model B.



Figure 8. Evolution of pr value on network structure generated after rewiring BA network with model B.

On the other hand, negative β can also influence the value of *pr*. However, its influence is two-fold: it is very limited on the structures generated from RG network while much wider on ER, WS and BA networks with the proposed rewiring strategies. As a specific example, when β =-2, the value of pr was promoted to only 0.05% on RG structure while it was 89.9%, 81.8%, 51.8% on ER, WS and BA, respectively. This implies that structures generated by RG model have a little information spreading influence with the proposed rewiring strategies when considering negative β . This may indicate that individuals are sometimes less responsive to the payoff differences, and an individual with a large payoff may rewire to individual with small payoff [7]. To this end and in order to provide further credence lets recall the results reported on a recent study in [14].

3.1.3. Results of the Model in [14]

Below, we recall the results of network structures generated by the model in [14]. These results underline

the hypothesis that network structure and parameter β have a great influence in information spreading. Evidently, when $\beta=2$, the *pr on* RG, ER, WS and BA networks was promoted to 0.097%, 21%, 17.5% and 23.75%, respectively. Also, the structures of these networks generated by static network technique were found to produce *pr* of 0.049%, 2.5%, 1.25% and 18.75%, respectively. On the other hand, the influence of *pr* evolution was also found to depend on the value of β . For example, the values of *pr* on BA network obtained for $\beta=2$, 0 and -2 were approximately 23.75%, 18.71% and 2.3%, respectively. A similar case was observed on ER network where the values of *pr* were promoted to 21%, 7.5% and 1.25% when considering $\beta=2$, 0 and -2, respectively.

Table 1. Summary on values of pr (approximated) generated by various network structures with several techniques.

Networks	Techniques	Model A	Model B	Model in [14]
RG	2	36%	58.3%	0.097%
	0	10.4%	42.6%	0.069%
	-2	0.04%	0.05%	0.0625%
	Static	0.05%	0.05%	0.049%
ER	2	80.5%	85.1%	21%
	0	85%	89.3%	7.5%
	-2	86%	89.9%	1.25%
	Static	80%	80%	2.5%
WS	2	72.1%	79.4%	17.5%
	0	73.2%	81.7%	4.9%
	-2	72.5%	81.8%	1.25%
	Static	63.6%	63.6%	1.25%
BA	2	29.8%	38.1%	23.75%
	0	26.5%	38.8%	18.71%
	-2	28.8%	51.8%	2.3%
	Static	18.1%	18.1%	18.75%

Besides, the results of the present models (models A and B) suggested that random network promotes more efficient information spreading compared to the other networks. On the contrary, recently Liu and Zhang [14] reported that scale free network is a vital promoter of information spreading. On the other hand, small world network demonstrated more effective spreading than scale free and random networks [29]. To this end, we have shown how information spreading is highly sensitive to both network structure and parameter β . A clear distinction of the effects of parameter β and a variety of network structures generated by the above mentioned models are summarized in Table 1.

3.2. Results of Acquainted Nodes (*pi*) Obtained from Various Network Structures

Let us further examine the role of network structure and parameter β in information spreading by another set of experiments. This time the focus was on studying the number of acquainted nodes (*pi*) in each time step. The same network structures generated by the models were considered. In general, a larger value of *pi* signifies more spreading of information in a single time step. Figures 9, 10, 11, and 12 illustrate the evolution of pi obtained after rewiring the networks with Model A. It was found that for the same value of β , various network structures influence different ranges of pi. For example, when $\beta=2$; the value of pi on structures generated from RG, ER, WS and BA were promoted to 3.92%, 23.5%, 18.6% and 6.84%, respectively.



Figure 9. Evolution of pi value on network structure generated after rewiring RG network with model A.



Figure 10. Evolution of pi value on network structure generated after rewiring ER network with model A.



Figure 11. Evolution of pi value on network structure generated after rewiring WS network with model A.



Figure 12. Evolution of pi value on network structure generated after rewiring BA network with model A.

3.2.2. Results of The Model B

The network structures obtained after rewiring the networks with Model B have shown a behavior similar to the case of model A. Different network structures enhance pi to different ranges. Evidently, as shown in Figures 13, 14, 15, and 16, the value of pi on the structures obtained from RG, ER, WS and BA were promoted to 13.3%, 29.3%, 25.94% and 9.58%, respectively when $\beta=2$.



Figures 13. Evolution of *pi* value on network structure generated after rewiring RG network with model B.



Figure 14. Evolution of pi value on network structure generated after rewiring ER network with model B.

Moreover, for one particular network structure, the value of *pi* was promoted to various ranges when different values of parameter β were set in the experiments. For instance, when the value of β on WS network was set to 2, 0 and -2, the pi was enhanced to 25.94%, 22.11% and 16.87%, respectively. More excitingly, BA network revealed that when $\beta=2$, the value of *pi* increases rapidly in the early stages and suddenly drops to zero. At the same time it continues to increase in several steps for β =-2 before gradually dropping to zero, as in the case of Model A but this time much wider ranges of *pi* were observed. Additionally, in both Figures from 9 to 16 there is a remarkable difference in ranges of *pi* between $\beta = 2$ and static network, in a good agreement with the simulation results in Figures from 1 to 8 and also the study in [14].

Table 2. Summary on values of pi (approximated) generated by various network structures with several techniques.

Networks	Techniques	Model A	Model B	Model in [14]
RG	2	3.92%	13.3%	0.028%
	0	0.33%	5.2%	0.026%
	-2	0.01%	0.01%	0.025%
	Static	0.01%	0.01%	0.024%
ER	2	23.5%	29.3%	3%
	0	22.5%	29.1%	0.5%
	-2	18.1%	25.37%	0.125%
	Static	15.8%	15.84%	0.25%
WS	2	18.6%	25.94%	2.4%
	0	14.7%	22.11%	0.4%
	-2	10.5%	16.87%	0%
	Static	8.28%	8.28%	0%
BA	2	6.84%	9.58%	8.1%
	0	3.75%	6.24%	6%
	-2	1.5%	6.08%	0%
	Static	2.61%	2.61%	6%

3.2.3. Results of The Model in [14]

The results in their model revealed that RG, ER, WS and BA networks produces pi to 0.028%, 3%, 2.4 % and 8.1%, respectively when β =2. On the other hand, the values of pi generated by various

values of β were different on the same network structure. For example, the values of *pi* on ER network were 3%, 0.5% and 0.125% when the value of β was2, 0 and -2, respectively. The results shown in Figure 1 to Figure 16 and the results in the study by Liu and Zhang [14] provide evidence to conclude that network structure and both positive and negative β hold an important role in information spreading. The results are summarized in Table 2.

4. Summary and Concluding Remarks

We have studied the effects of network structures and dependency of parameter β in enhancing information spreading on dynamic social networks by considering different rewiring strategies. It is widely known that network structure promote information spreading [26, 29]. We have shown, however, that network structure is a'two-edged sword' for information spreading: it can promote high levels of information spreading for some values of parameter β , but other values can cause spreading influence to plummet. For example, the structures of RG with negative β (β =-2) can prevent information spreading, but about 0.05 per cent of the population can be informed. Further, network structures generated by the Model B promote more effective spreading in most cases than the structures generated by the other models. This implies that the connectivity fluctuations of a network play a major role by strongly enhancing the spreading process. Furthermore, a society of dynamic interaction is significantly more efficient in information spreading than the one with static interactions. This is consistent with the studies in [14, 16].

Moreover, negative β can influence high ranges of information spreading on random network, small world network and scale free network depending on the strategy used to generate their structures. More specifically, scale free network generated by the model B clearly proves much wider range of information spreading with negative β than with positive β . Additionally, the enhancement of information spreading on random network is more efficient than the other networks when considering the model B. As a closing remark, the simulation results also give a good indication that rewiring more than three edges of random network could yield no significant advantages in promoting information spreading.

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References

- [1] Albert R. and Barabási A., "Statistical Mechanics of Complex Networks," *Reviews of Modern Physics*, vol. 74, no. 1, pp. 47-97, 2002.
- [2] Anderson R. and May R., *Infectious Diseases of Humans*, Oxford University Press, 1992.
- [3] Bailey N., *The Mathematical Theory of Infectious Diseases and Its Applications*, Hafner Press, 1975.
- [4] Barabási A. and Albert R., "Emergence of Scaling in Random Networks," *Science*, vol. 286, no. 5439, pp. 509-512, 1999.
- [5] Du W., Zheng H., and Hu M., "Evolutionary Prisoner's Dilemma on Weighted Scale Free Networks," *Physica A: Statistical Mechanics and its Applications*, vol. 387, no. 14, pp. 3796-3800, 2008.
- [6] Fowler J. and Christakis N., "Cooperative Behavior Cascades in Human Social Networks," *The National Academy of Sciences*, vol. 107, no. 12, pp. 5334-5338, 2010.
- [7] Fu F., Rosenbloom D., Wang L., and Nowak M., "Imitation Dynamics of Vaccination Behavior on Social Networks," *The Royal Society B: Biological Sciences*, vol. 278, no. 1702, pp. 42-49, 2011.
- [8] Granovetter M., "The Strength of Weak Ties," *American Journal* of *Sociology*, vol. 78, no. 6, pp. 1360-1380, 1973.
- [9] Hethcote H., "The Mathematics of Infectious Diseases," *Society for Industrial and Applied Mathematics*, vol. 42, no. 4, pp. 599-653, 2000.
- [10] Ichinose G. and Kobayashi M., "Emergence of Cooperative Linkages by Random Intensity of Selection on a Network," *BioSystems*, vol. 105, no. 1, pp. 1-9, 2011.
- [11] Kef M., Chergui L., and Benmohammed M., "Self-organization and Topology's Control for Mobile Ad-hoc Networks," *The International Arab Journal of Information Technology*, vol. 8, no. 3, pp. 227-234, 2011.
- [12] Kempe D., Kleinberg J., and Tardos E., "Maximizing the Spread of Influence Through a Social Network" in Proceeding of the 9th ACM International Conference on Knowledge Discovery and Data Mining, Washington, pp.137-146, 2003.
- [13] Li T. and Xiao N., "Solving QBF with Heuristic Small World Optimization Search Algorithm," *The International Arab Journal of Information Technology*, vol. 12, no. 4, pp. 370-378, 2015.
- [14] Liu C. and Zhang Z., "Information Spreading on Dynamic Social Networks," *Communications in*

Nonlinear Science and Numerical Simulation, vol. 19, no. 4, pp.896-904, 2014.

- [15] Lü L., Chen D., and Zhou T., "The Small World Yields the Most Effective Information Spreading," *New Journal of Physics*, vol. 13, no. 12, pp. 123-005, 2011.
- [16] Miritello G., Moro E., and Lara R., "Dynamical Strength of Social Ties in Information Spreading," *Physical Review E*, vol. 83, no. 4, pp. 1-4, 2011.
- [17] Moreno Y., Pastor-Satorras R., and Vespignani A., "Epidemic Outbreaks in Complex Heterogeneous Networks," *The European Physical Journal B*, vol. 26, no. 4, pp. 521-529, 2002.
- [18] Perc M. and Szolnoki A., "Co-evolutionary Games-A Mini Review," *Biosystems*, vol. 99, no. 2, pp. 109-125, 2010.
- [19] Rosenschein J. and Zlotkin G., *Rules of Encounter*, MIT Press, 1994.
- [20] Segbroeck S., Pacheco J., Lenaerts T., and Santos F., "Emergence of Fairness in Repeated Group Interactions," *Physical Review Letters*, vol. 108, no. 15, pp. 1-5, 2012.
- [21] Szabó G. and Toke C., "Evolutionary Prisoner's Dilemma Game on a Square Lattice," *Physical Review E*, vol. 58, no.1, pp. 1-7, 1998.
- [22] Traulsen A., Pacheco J., and Nowak M., "Pairwise Comparison and Selection Temperature in Evolutionary Game Dynamics," *Journal of Theoretical Biology*, vol. 246, no. 3, pp. 522-529, 2007.
- [23] Tuyls K. and Parsons S., "What Evolutionary Game Theory Tells us about Multi-agent Learning," *Artificial Intelligence*, vol. 171, no. 7, pp. 406-416, 2007.
- [24] Volz E. and Meyers L., "Epidemic Thresholds in Dynamic Contact Networks," *Journal of The Royal Society Interface*, vol. 6, no. 32, pp. 233-241, 2009.
- [25] Watts D. and Strogatz S., "Collective Dynamics in Small World Networks," *Nature*, vol. 393, no. 1, pp. 440-442, 1998.
- [26] Watts D., Small Worlds: The Dynamics of Networks Between Order and Randomness, Princeton University Press, 1999.
- [27] Wu B., Zhou D., Fu F., Luo Q., Wang L., and Traulsen A., "Evolution of Cooperation on Stochastic Dynamical Networks," *Public Library* of Science, vol. 5, no. 6, pp. 1-7, 2010.
- [28] Yamauchi A., Tanimoto J., and Hagishima A., "What Controls Network Reciprocity in the Prisoner's Dilemma Game," *Biosystems*, vol. 102, no. 2-3, pp. 82-87, 2010.
- [29] Zhang Z., Zhang C., Han X., and Liu C., "Emergence of Blind Areas in Information Spreading," *Public Library of Science*, vol. 9, no. 4, pp. 1-11, 2014.



Abdulla Ally received his Bachelor of Science in Computer Science in 2004 and Master of Science in Computer Science in 2008, both from University of Dar es Salaam, Tanzania. Abdulla is currently a Ph.D. candidate in Systems Analysis

and Integration at the University of Shanghai for Science and Technology, China. He has participated in several research projects in computer science domain for more than 7 years. His research interests revolve around, but not limited tocomplex networks, modelling of networks and dynamic processes (such as information spreading), graph theoryandinformation systems.



Ning Zhang received her B.E. degree in Wireless Communication from the Nanking Institute of the Posts and Telecommunications, China in 1982 and M.E. degree in Systems Engineering from East China University of Technology,

China in 1991.From 1982 to 1984, she was an assistant engineer with the Shaanxi Wireless Communication Bureau, Xian, China. Currently, she is a Professor in the Business School at the University of Shanghai for Science and Technology (USST), China. Before joining the USST, she was an Assistant Professor at the Jiangsu Institute of Technology, China. Her research interests include complex network, network model, and graph theory.