

Fingerprint Verification Methods Using Delaunay Triangulations

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Abstract: *This paper presents a modification for robust minutiae based fingerprint verification methods that use Delaunay triangulations. The purpose of this modification is to decrease the number of comparison operations and the error rates within the matching process, by doing a full analysis of the Delaunay triangles. From this full analysis, a modified method was proposed. The identified minutiae represent nodes of a connected graph composed of triangles. With this technique, the minimum angle over all triangulations is maximized, which gives local stability to the constructed structures against rotation and translation variations. Geometric thresholds and minutiae data were used to characterize the triangulations created from input and template fingerprint images. The effectiveness of the proposed modification is confirmed with calculations of False Acceptance Rate (FAR), False Rejected Rate (FRR) and Equal Error Rate (EER) over FVC2002 databases compared to other approaches results.*

Keywords: *Angle of orientation, delaunay triangulation, EER, fingerprint, geometric thresholds.*

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1. Introduction

Although, authentication systems based on biometrics have numerous advantages over conventional security systems, the biometric data acquired from the attributes of human body is noisy by nature [13, 14].

The quality of fingerprint image is normally affected by three types of degradations: appearance of gaps between ridges, parallel ridges interceptions and natural effects such as cuts, wrinkles and injuries. Image enhancement processes are in charge of improve the contrast between ridges and valleys, as well as reduce the noise in the image [8, 10].

After enhancing the fingerprint image, comes the process of extracting and matching fingerprint features, which can be classified in three categories: based on minutiae [5, 9, 11], based on image [6, 7, 14] and hybrid [1, 8, 12]. Those ones based on minutiae, use a feature vector extracted from fingerprints as a set of points in a multi-dimensional plane. After the extraction process, the fingerprint matching becomes a non-rigid point-matching problem with unknown correspondences and differences in the number of points belonging to two sets (query and template). Moreover, the skin elasticity changes the relative position of the minutiae at each acquisition [11] and can cause that genuine minutiae get lost or pseudo minutiae appear [9].

For fingerprint verification processes, Delaunay Triangulation can be formed if the minutiae locations are taken as the set of points P . The advantages of this proposed method are: Every minutiae keeps the same neighboring structure even in presence of distortion.

Insertion of new points in the triangulations because of noise affects only locally. The same happens with missing or spurious minutiae. Using geometrical thresholds, each triangle in the Delaunay Triangulation can be characterize in a unique way. Equal Error Rate of less than 1% can be obtained using these techniques, demonstrating the accuracy of the method under these thresholds.

The novelty of our contribution respect to other approaches is in the full analysis of the Delaunay Triangulations to compare minutiae structure between fingerprints. Most minutia based approaches that use Delaunay Triangulations, start their analysis by studying the edges that form Delaunay triangles instead of analyzing these triangles as a basic structure [5, 9, 11]. In some cases, even a threshold is established to decide whether to use Delaunay Triangulations or process all possible combinations of minutiae forming an edge between them [5]. This kind of analysis involves more computational load because of the number of combinations to analyze and the operations required for each comparison. Furthermore, most of these approaches perform a fingerprint alignment based on one reference triangle. To find this reference, an analysis of triangle's edges is done using specific thresholds. Then after the alignment, other thresholds have to be established to define a neighborhood where aligned minutiae must be. With this, several alignments and comparisons have to be executed in order to count the matching aligned minutiae, calculate a matching score and find the best alignment [9, 11].

In our proposal, from the beginning, every Delaunay triangle extracted from the query fingerprint is compared against every triangle extracted from the template fingerprint. In this comparison, the analysis of the three edges of the triangle and the three vertices of the triangle are performed. To avoid fingerprint alignment operations, a measurement and a comparison of the orientation of triangles is performed and by establishing an specific threshold, the possible variation of fingerprint rotation is considered. In the case of spatial displacement of the fingerprints, the structure of the Delaunay Triangulation is tolerant because it keeps the same association of the nearest minutiae to represent each triangle, when the distortion affecting the fingerprint image is uniform. Finally, the matching score is calculated as the rate of coincident triangles over the average of the total number of minutiae between both fingerprints.

This paper is organized as following: section 2 describes related work around the problem of fingerprint verification. Section 3 explains the mathematical and computational theory behind Delaunay Triangulations and the measurements calculated to characterize a fingerprint. Section 4 details our proposed solution and how our scheme was implemented. Section 5 shows the results obtained after applying the proposed solution to 4 FVC2002 databases. Then a discussion of these results is presented. Finally, section 6 contains the conclusions of this work in terms of the results obtained and the future work to improve the performance of the scheme proposed.

2. Related Works

2.1. Image based Approaches

Among image based methods for fingerprint verification, there is a proposal based on features extracted from Wavelet and Fourier-Melling Transformation (WFMT). Wavelet transformation is used to preserve the local edges and reduce noise in the low frequency domain after image decomposition, which makes the fingerprint image less sensitive to shape distortion. Then Fourier-Melling Transformation (FMT) serve to produce a translation, rotation and scale invariant feature. The results obtained in [7], show that verification accuracy is 5.66 and Equal Error Rate (EER) of 1.01%.

A second proposal is the use of tessellated invariant moment features for fingerprint verification. Some intrinsic properties of the fingerprints are estimated, such as foreground region mask, local ridge orientation and local ridge frequency to enhance fingerprint image. By using different complex filtering methods, a reference point is established. Then its orientation is calculated using the least mean square orientation algorithm. A Region Of Interest (ROI) is centered on the reference point and tessellated in a predefined

number of square cells. Seven invariant moments are extracted from the cells and represent the fingerprint information of the local structure. The verification is based on measures of similarity to the Eigenvalue-Weighted Cosine (EWC) distance, to match two corresponding feature vectors. The experiments done over the FVC2002 4 databases show an average EER of 3.57% using the EWC distance [15].

2.2. Hybrid Approaches

One solution from hybrid fingerprint extraction and matching processes was proposed in [12] and uses both minutiae and ridge flow information. To capture the ridge strength at equally space orientations, a set of 8 Gabor filters in the spatial frequencies that correspond to the average inter-ridge spacing in fingerprints is used. Then an eight-dimensional ridge feature map is constructed with square tessellation of the filtered images. This map and the minutiae set of a fingerprint are used for matching purposes. For this scheme the EER calculated is about 4%.

2.3. Minutiae based Approaches

About proposals in the minutiae based category for fingerprint matching, there are few researches that utilize Delaunay Triangulations to compare a query and a template fingerprint image. In 2004 Parziale, and Niel, proposed to establish the dependency among minutiae by applying Delaunay Triangulation over the point set representing them. In that structure, each minutia was used as a triangle's vertex. Then measures of distance between minutiae pairs, angular difference between orientations of minutiae pairs and angles between the orientation of each minutia and the segment connecting them with another minutia, were calculated.

After applying three geometric filters, some triangulations in the query set were selected as candidates for the matching with the triangles of the template set. For those, an alignment procedure is made with the triangulations in the query set, in terms of spatial coordinates and angles of orientation. If a minutia in the transformed query set is close enough to a minutia in the template set, it is counted and used later to calculate a matching score [11].

Liu *et al.* [9] proposed a very similar fingerprint matching algorithm based on Delaunay Triangulations to find Reference Minutiae Pairs Known as (RMPs). The analysis begins with similar edge pairs formed from query and template sets of minutiae. Measurements of Euclidean distance, minutia orientation and edge orientation are compared after that. If a pair of edges is very similar, then the triangles to which they belong become candidates for the next analysis phase. In the second phase, distance of triangle's sides and internal angles are compared. If the coincidence between triangles exists, for each

triangle an alignment of the query set with the template set is done, using geometrical equations. Finally for each alignment of points a counting is done and a matching score is calculated [9].

The last proposal related to the use of Delaunay Triangulations, was made by Deng and Huo [5]. Instead of finding the best-matching minutiae pairs, the objective was to find the best-matching edge pairs. Other important changes were: Minutiae orientation was mapped to a range from 0 to 2π instead of using the original from 0 to π and the use of minutiae type as a parameter to compare. The ridge count between minutiae was also another data used in the matching process.

The matching process starts by checking the number of minutiae in the fingerprint image, if it is below a threshold, Delaunay triangulations are not calculated and instead all the possible edges connecting two minutiae are considered. Then, several geometric filters are applied to the edge pairs, like Euclidean distance, minutiae orientation difference, among others. The minutiae of the edges that satisfy those filters are used in the next phase. The remaining minutiae are sorted in ascending order and form all the possible triangles with the closest selected neighbor minutiae. Later, geometric comparisons between triangles are performed and a matching score is calculated for each triangle. At the end, a second matching score between the query and template image is calculated considering the previous triangle matching scores [5].

In later sections of this paper, some of these different schemes will be re-taken in a comparison against our proposed minutiae-based solution. Then a discussion about the obtained results in terms of the EER, False Acceptance Rate (FAR) and False Rejected Rate (FRR) thresholds will take place.

3. Background

A Triangulation can be defined as the maximal planar subdivision whose vertex set is P , where P denotes a finite set of points in a plane. What a maximal planar subdivision means is a subdivision such that no edge connecting two vertices can be added to this subdivision without destroying its planarity (any edge that is not in the subdivision intersects one of the existing edges).

“Let P be a set of points in the plane, and let T be a triangulation of P . Then T is a Delaunay triangulation of P if and only if the circumcircle of any triangle of T does not contain a point of P in its interior” [3].

So any Delaunay Triangulation T maximizes the minimum angle over all triangles that compose it.

To start building a Delaunay Triangulation with only the set of points P as initial data, it is required to create an initial repository. From a geometrical point of view, the repository is the first triangle in the

Delaunay Triangulation and it is large enough to contain the whole set of points P . The vertices of the first triangle are three extra points, p_0 , p_{-1} and p_{-2} . It is important to choose p_0 , p_{-1} and p_{-2} far enough away, so they do not destroy any triangles in the Delaunay triangulation of P . Later p_0 , p_{-1} and p_{-2} can be discarded together with all their incident edges.

About the existing algorithms to implement a Delaunay Triangulation, there are basically 2 types: the first type is a static algorithm where the triangulation is valid after every single point is processed. Some examples are: The recursive split algorithm, the divide and conquer algorithm, the step by step algorithm, the modified hierarchical algorithm, among others.

The second type of algorithms is the dynamic triangulation where the triangulation is valid during processing. This makes possible to view the contribution of one point to the Triangulated Irregular Network (TIN). Some algorithms to implement this are called incremental as the Bowyer and Watson algorithm.

Bowyer and Watson algorithm is known as a incremental delete and build algorithm because it adds points sequentially into an existing Delaunay triangulation [2]. The process follows the next steps: for each point in set P :

1. Insert point $p \in P$ into triangulation.
2. Find all existing triangles whose circumscribing circle contains the point p .
3. All triangles found in step 2 are deleted and a convex cavity is created.
4. The point p is joined with all the vertices on the boundary of the cavity formed in 3 (re-triangulation).

Finally, the implementation of the Bowyer-Watson algorithm requires computational data structures such as: point based data structure for the vertices of the triangle, a triangle based data structures for the elements that compose the Delaunay Triangulation and finally a directed acyclic graph that represents the Triangulation with the edges of each triangle and the neighbors that shares them.

For the implementation of Delaunay Triangulations in our proposal, the Bowyer and Watson algorithm was used because it provides the same theoretical optimum algorithmic complexity that other methods, which is $\Theta(N \log_2 N)$, but with an easier procedure. Although Delaunay Triangulations allow to create a rotation and distortion tolerant structure, the characterization, identification and comparison between two different triangulations must be done manually. Therefore, the second part of our proposal consists of a set of measures and geometric thresholds that allow us to distinguish each triangle and its vertices in the Delaunay triangulations formed.

4. Proposed Modified Method

The proposed scheme is a minutiae based fingerprint verification system that uses Delaunay Triangulations with geometrical measurements and thresholds to validate the similitude between two different fingerprints.

- *Stage 1.* Fingerprint image capture
 - a. Use of Griaule Biometrics Fingerprint SDK 2009.
- *Stage 2.* Minutiae features extraction.
 - a. Create a feature extraction vector in ANSI 278-2004 format.
- *Stage 3.* Delaunay Triangulation creation.
 - a. Create a semi infinite reference triangle.
 - b. Insert a new point in the Cartesian plane.
 - c. Search the triangle containing the new point inserted.
 - c.1. Check if point was already inserted.
 - c.2. If not, calculate the Cross product between the vertices of the triangle containing the point inserted and the point itself.
 - d. Determine the Cavity produced by the new point in the Delaunay Triangulation.
 - d.1. Verify if new point is inside, is on, or if it is outside of a Triangle's circumcircle and its neighbors circumcircles.
 - d.2. If new point is inside a circumcircle of some triangle, add it to cavity list.
 - e. Update Delaunay Triangulation.
 - e.1. Remove triangles contained in the cavity list calculated from Delaunay Triangulation.
 - e.2. Create new triangles with remaining vertices around the old cavity and the new point inserted.
 - e.3. Add new Triangles to Delaunay Triangulation.
 - e.4. Update each link between adjacent triangles.
- *Stage 4.* Triangle's Data Structure creation.
 - a. Calculate Triangle's internal angles.
 - b. Calculate Euclidean distance between triangle's vertices.
 - c. Calculate Triangle's angle of alignment with x axis.
 - d. Calculate the difference between each pair of vertex's angle and triangle side slope.
- *Stage 5.* Triangle features comparison.
 - a. Align a pair of triangles based on internal angles.
 - b. Compare a pair of triangle's vertices based on minutiae type.

- c. Compare arithmetically, the length of the sides of a pair of triangles.
- d. Compare arithmetically the angular differences between each pair of vertex angle and the slope of a triangle side that connects with that vertex, for a pair of triangles.
- e. Compare arithmetically the angle of alignment with x axis of a pair of triangles.
- f. Authenticate or reject the fingerprint analyzed.

In each comparison of the stage 5, the corresponding threshold for error tolerance is added.

In stage 2, the feature vector extracted using the software of Griaule Biometrics for each minutia, contains the following biometric data:

$$m_i = \{x, y, \theta, \tau\} \quad (1)$$

Where:

- $m_i = i^{\text{th}}$ minutia.
- $x =$ value of the spatial coordinate in the x -axis.
- $y =$ value of the spatial coordinate in the y -axis.
- $\theta =$ minutia orientation $[0, 180^\circ]$.
- $\tau =$ type of minutia [end of ridge, ridge bifurcation, other].

For stage 5, different measures are calculated in order to uniquely characterize each triangle. The first measure, related to alignment of triangles can be resumed in Equation 2, which comes from the cosine law for triangles.

$$\gamma_i = \cos^{-1} \left(\frac{d_{ij}^2 + d_{jk}^2 - d_{ki}^2}{2d_{ij}d_{jk}} \right) \quad (2)$$

Where:

- $\gamma_i =$ triangle's internal angle.
- $d_{ij}, d_{jk}, d_{ki} =$ length of triangle's side.

An alignment of triangles is needed to ensure that, the order in which the vertices of two triangles in different Delaunay Triangulations is described, is the correct one. Otherwise, any further calculation will be meaningless because the vertices of the triangles could be swapped. To achieve this, the internal angles of a pair of triangles are paired in a way that there exists the minimum difference possible between them.

When a pair of angles has the smallest difference, the vertex of the triangle containing one of the angles is renamed so that it matches with the vertex's name in the other triangle. The process is repeated until the three vertices are matched with the best option in the second triangle. Once a pair of triangles is aligned, a process of comparison between them has place.

The first test checks the type of minutiae in the vertices with the same name in the different triangles. The types of minutiae are classified as: Termination, bifurcation or others. If the first test is passed, the difference of one triangle's vertex angle (minutia orientation) and the angle of the segment connecting

that vertex with another one is calculated, for each of the vertices of a triangle. Equation 3 describes how to calculate this measure. A threshold expressed in grades is established to allow a tolerance limit in the rotation that a triangle can have over another.

$$a_{ij} = \tan^{-1} \left(\frac{y_i - y_j}{x_i - x_j} \right) - \theta_i \quad (3)$$

Where:

- a_{ij} =angle difference between triangle's side slope expressed as an angle and minutia orientation.
- y_i, y_j =y-spatial coordinate of minutiae i and j respectively.
- x_i, x_j =x-spatial coordinate of minutiae i and j respectively.
- θ_i = i 's minutia orientation.

The next comparison is done with Euclidean distance Equation 4, between the vertices of a triangle. Again a threshold is established, this time in pixels, to tolerate a certain grade of distortion in the shape of the triangle because of different spatial allocation of the vertices.

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (4)$$

Where:

- d_{ij} = Euclidean distance between minutiae i and j .
- x_i, x_j = x-spatial coordinate of minutia i and j .
- y_i, y_j = y-spatial coordinate of minutia i and j .

The final filter is related to Equation 5 which evaluates the angle between the first vertex of a triangle and the x axis. A threshold expressed in pixels allows a tolerance degree of rotation for the triangle being evaluated in case the fingerprints captured are rotated. It is important to point out that this measure quantifies the degree of rotation that a local area of the fingerprint has in terms of triangles that belong to a Delaunay Triangulation. With this, an alignment of all minutiae in the fingerprint is no longer needed, since if there is a rotation of the fingerprint, every minutia and every triangle composed of them will have the variation and it will fit in the threshold established.

$$\delta_i = \cos^{-1} \left(\frac{V_{ref} \cdot d_{ki}}{|V_{ref}| |d_{ki}|} \right) \quad (5)$$

Where:

- δ_i =angle between one triangle's side ij and V_{ref} which is a vector parallel to x -axis.
- V_{ref} =vector from triangle's minutia m_i to y -axis, which is parallel to x -axis.
- d_{ki} =Euclidean distance between minutiae k and i .

At the end if each of the previously mentioned test are passed successfully then the system recognizes both triangles as the same one and increases the count of equal elements between both fingerprint's Delaunay Triangulations. Finally, the percentage of coincidence

between both fingerprints can be calculated as stated in Equation 6.

$$\% = \frac{t_{equals}}{\frac{t_p + t_c}{2}} \cdot 100\% \quad (6)$$

Where:

- t_{equals} =number of triangles considered as equal in both Delaunay Triangulations.
- t_p =Total number of triangles in the Delaunay Triangulation of the biometric template.
- t_c =Total number of triangles in the Delaunay Triangulation of the query fingerprint image.

The last important aspect to discuss is the setting process for the geometric thresholds used in the previous comparisons. Most of the methods that implement and use Delaunay Triangulations, set the geometric thresholds empirically. In our proposal this can be set manually or it can be calibrated by a process similar to an enrollment.

The process begins by extracting the minutiae from a first fingerprint image, calculating the Delaunay Triangulation representation, and getting a first group of empiric geometric thresholds for the comparison. Then, a second fingerprint image is processed, extracting its minutiae and creating its Delaunay Triangulation.

After that, a matching process as the one described before has place. A comparison of minutiae type, length of triangle's sides, angular difference between minutiae orientation and triangle's side slope, and triangle's angle of orientation is executed. For each one of the last three comparisons, and for each of the triangles in both Delaunay Triangulations, an average of the difference between these characteristics is calculated. Then, the percentage of coincidence between the fingerprint images is processed using Equation 6.

If only two fingerprint images were used in this process of calibration, and if the percentage of coincidence between them is less than 10%, then, the averages previously calculated are used as new geometric thresholds. If more than two fingerprint images were enrolled, and if the percentage of coincidence between them is more than 0% but less than 10%, then the geometric thresholds are increased by 25% each time. This process is repeated for each new fingerprint image used in the calibration.

Equal Error Rates below 1% can be obtained when using these techniques, which is a very good indicator for biometric verification systems.

5. Experiment

A graphical proof that describes how Delaunay Triangulations act as an effective filter for fingerprint images, can be seen in the following 3 scenarios, represented with different graphics. In the first

scenario two samples of the same fingerprint were taken, in this case almost no distortion, translation, rotation or change of pressure occurred in both fingerprints. The Delaunay Triangulation constructed from this images were:

As it can be seen the Triangulations calculated in Figure 1-c in the left is almost the same as the triangulation calculated in Figure 1-e. The graphic in the middle, Figure 1-d shows both triangulations overlapped and this is where it can be appreciated that only 3 minutiae are different. One of them appear only in the triangulation shown in Figure 1-c and the other in triangulation in Figure 1-e. The last one, is displaced a few pixels to the right.

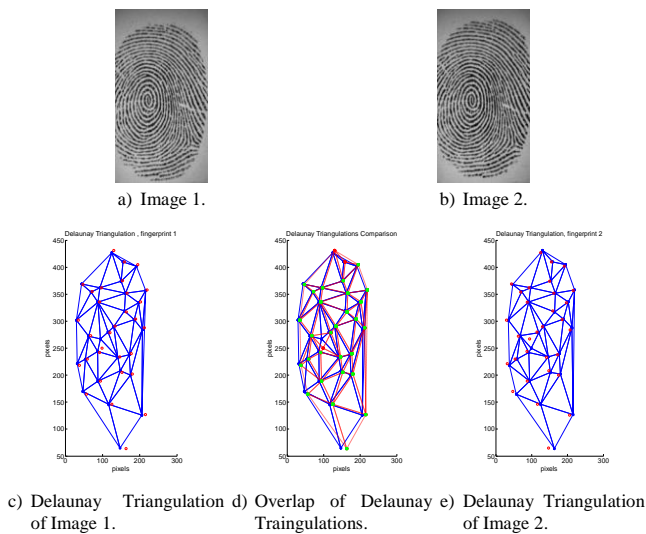


Figure 1. Scenario, two samples very similar of the same fingerprint were taken.

The second scenario shows another two samples from the same fingerprint, but in this case a change in pressure levels and a small rotation were registered. Again a group of graphics with the Delaunay Triangulations were processed. The results are presented in Figure 2.

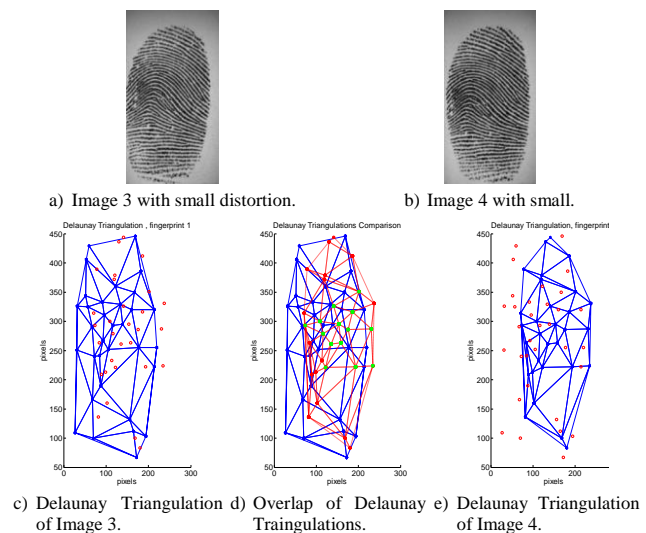


Figure 2. Second scenario, a change in pressure levels and a small rotation was registered.

From the graphics of Figure 2 it can be seen that Delaunay Triangulations differs one from another but also there are specific areas in the Triangulations that contain some triangles with the same characteristics in both: Graphic in Figure 2-c and e. In the graphic of Figure 3-d in a softer color some of the vertices that appear in both triangulations are marked. This proves that Delaunay Triangulations can characterize fingerprints even if they differ because of displacement, distortion, rotations, changes of pressure or other types of alterations.

The third and final scenario shows two samples of two different fingerprints. Because of this it is expected that none triangle in both Delaunay Triangulations calculated would be classified as equal. This is the most difficult task, because small triangles tend to be very similar in dimensions and angles respect to others. Figure 3 shows this.

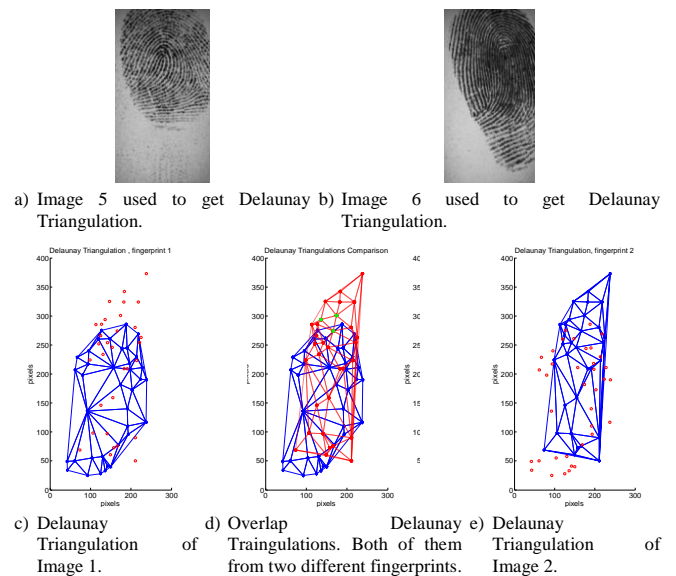


Figure 3. Third scenario, two samples of two different fingerprints.

From the graphics of Figure 3 it can be seen that the calculated Delaunay Triangulations are totally different. Because of that only one triangle is common between both triangulations. It is not easy to find where is the small triangle marked with a softer color in the graphic d). in the other two graphics. But it is clear that the triangle marked as equal is very small and easy to classified wrong.

It is very important to point out that for the experiments showed before, there were already taken in account the different geometric thresholds to filter all the triangles in the Delaunay Triangulations. Otherwise, if only a check up about triangles being present were done, almost none triangle will be equal to any other in the second triangulation. That can be seen easily from the graphics.

After testing the advantages of using our modified method with some real fingerprint images and effectively characterize their minutiae, we amplified the number of tests in order to get the FAR and FRR

thresholds to compare our scheme with other proposals. To get such thresholds, we consider 4 fingerprint databases from FVC2002 set A, which are summarized in Table 1. Our results were compared against obtained results of the 3 different proposals reviewed in the related work section of this paper.

Table 1. FVC2002 Databases characteristics [4].

	Sensor Type	Image Size	Set A(wide x deep)	Resolution
DB1	Low Cost Optical Sensor	300 x 300	100 x 8	500 dpi
DB2	Low Cost Capacitive Sensor	256 x 364	100 x 8	569 dpi
DB3	Optical Sensor	448 x 478	100 x 8	500 dpi
DB4	Synthetic Generator	240 x 320	100 x 8	About 500 dpi

For each fingerprint, 6 random images out of 8 were taken for storage, and the remaining 2 were used for verification test purposes. This means that in each database there were 600 storage images and 200 test images. The selection was repeated 4 times with different test images each time. The ROC curve for each fingerprint database is calculated based on the previous FAR and FRR data. Figures 4, 5, 6 and 7 show the ROC curves.

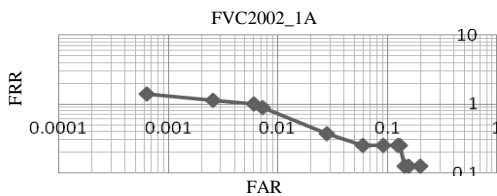


Figure 4. FVC2002_1A ROC curve.

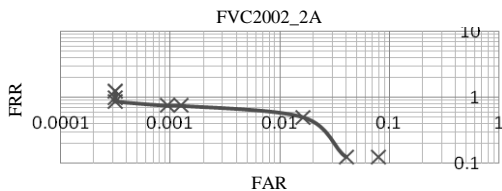


Figure 5. FVC2002_2A ROC curve.

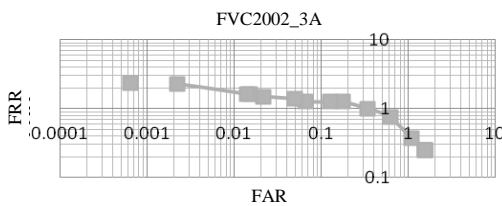


Figure 6. FVC2002_3A ROC curve.

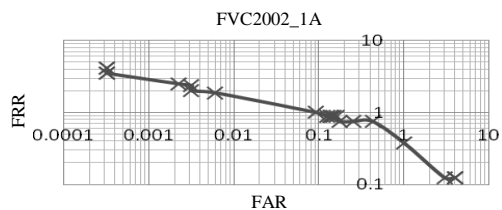


Figure 7. FVC2002_4A ROC curve.

6. Results

After analyzing the graphics presented before, the ERR was calculated and compared to other 3 proposed

schemes. Table 2 shows the summary of the comparison:

Table 2. EER from FVC2002 databases [8].

Method	DB_1A	DB_2A	DB_3A	DB4_A	Average
Yang <i>et al.</i> [9]	1.63	3.78	4.20	4.68	3.57
Beng <i>et al.</i> [8]	2.43	4.41	5.18	6.62	4.66
Ross <i>et al.</i> [11]	1.87	3.98	4.64	6.21	4.17
Deng <i>et al.</i> [4]	1.82	1.52	4.94	2.29	2.64
Our scheme	0.125	0.125	0.75	0.75	0.44

As can be seen, the performance of the proposed modified method exceeds that of those presented in the related work of this article. The difference between the average percentage is about 7 times smaller in our proposal. The main reasons why this happens are related to the type of post processing stage in which our scheme has place.

Instead of having a dispersion of points in a Cartesian plane, the analysis of minutiae takes place over a polygonal network that minimizes distortion and displacement of the vertices that compose them after the image preprocessing and enhancement have been done. With this, the use of spatial location of minutiae in the fingerprint is avoided, which is very important information to create fake fingerprints. And the most important thing, is that there is no need to calculate a single reference point as the core of the fingerprint, to center the analysis on it as it happens in [5, 6, 9].

Also, the criteria established to decide whether a minutiae in a fingerprint corresponds to another minutiae in a second fingerprint, strongly depends on a second set of measures from that polygonal network. For example, the use of the angle of orientation for each triangle in the Delaunay triangulations, gives a spatial characterization that works even with the smallest triangles in the structure. The orientation, follows a global pattern of the fingerprint, and it forces each triangle to follow it. Because of that and to the maximization of the internal angles in a Delaunay Triangulation, it is very difficult to find two triangles in different spatial locations with the same angle of orientation that also share other characteristics as size, and angles. With this, there is no reason to have restrictions of usage with small triangles as those presented in [5] where small edges are discarded for the analysis.

Other variables that play an important role in the identity verification results are the geometric thresholds, used to identify the equal triangles between the Delaunay Triangulations of each fingerprints. Depending on how strict the verification is required to be and the fingerprint images characteristics (size and resolution), the thresholds can be reduced to allow minimum or maximum variation among the polygonal networks. For the tests presented in this paper a tolerance of 10 pixels in distance and 10 grades in angles were established.

7. Conclusions

In this paper, a modification of previous minutiae based fingerprint verification methods was proposed. The innovation of this method relies in the full analysis of Delaunay Triangulations and geometric thresholds to identify fingerprint minutiae even in fingerprint images with noise. Measures of Euclidean distance, angles of orientation and angles of rotation make possible to characterize triangles with its vertices composed from minutiae, avoiding processes of minutiae alignment, more comparison operations and matching score calculations. In this way, rotation and displacement tolerance is achieved within the verification process.

It's important to emphasize that a calibration of geometric thresholds must be done depending on the characteristics of the fingerprint images, like size, resolution, etc. With all these conditions achieved, a better EER can be obtained in the tests realized.

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