Parameter Optimization of Single Sample Virtually Expanded Method

Xiaoxia Zhao[1], Wenjun Meng[1], Jinhu Su[1], Yuxuan Chen[2]
[1]School of Mechanical Engineering, Taiyuan University of Science and Technology, China
[2]Design and Research Institute, North China Municipal Engineering Design and Research Institute Co. Ltd., China

Abstract: *Aiming at single sample experiment of sample number n=1, the relationship of parameters virtually expanded from n=1 to n=13 is derived in this paper，and the big-samples data are gained by Bootstrap method. Instead of existing methods, a developing particle swarm optimization based on Minimax is put forward. With the application of this method in the parameter optimization, the lower confidence limit approaches the lower confidence limit of the Semiempirical Evaluation Method with more rapid speed and higher precision. In this way, the most suitable augmented parameters virtually expanded from n=1 to n=13 are gained, which provides a better virtual augment method for the sample augment from n=1 to n=13.*

Keywords: *Bootstrap method, semiempirical evaluation method, single sample, augmented parameter, minimax, particle swarm optimization (PSO).*

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1. Introduction

According to the sampling theory, the larger the sample size of test is, the higher the evaluation accuracy will be, and thus the assessment can more truly reflect the quality of the product level [14, 19]. However, as for expensive test specimens, like heavy-duty machineries or missiles, only single sample test could be carried out. Bootstrap method is normally used to solve the test evaluation problems which test sample sizes n is larger than 10, and it could not be applied to the test evaluation of single simple, namely n=1. So how to determine the evaluation of single sample test with Bootstrap method is a subject of both theoretical meaning and great application value.

Bootstrap method is a widely used evaluation method [6, 11, 15]. Huang *et al*. [2] proposed the small scale sample test estimation method based on Bootstrap method [1]. Hu *et al*. [3] evaluated the missile accuracy under the extreme small sample test by the non parametric Bootstrap method and the parametric Bootstrap method. Kijewski and Kareem [7] assessed the quality of system identification to generate useful statistics and confidence intervals through a bootstrap approach. Kreiss and Paparoditis [6] is concerned with the application of the bootstrap to time-series data [5]. In articles, Bootstrap method is applied in different conditions.

In the paper, the Virtually Expanded Sample Estimation Method is proposed to virtually expand test samples from n=1 to n=13. Then the virtually expanded samples are evaluated by Bootstrap method. The estimation of confidence lower limit is also obtained, which can be compared with the confidence lower limit obtained by the semi-empirical evaluation method [16, 17].

Particle Swarm Optimization (PSO) has been widely studied and applied for its advantages of easy implementation, high precision and rapid convergence [10, 20]. Nickabadi *et al*. [9] present the first comprehensive review of the various inertia weight strategies of PSO reported in the related literature and proposed a new adaptive inertia weight approach. The hybrid algorithm that combined particle swarm optimization with simulated annealing behavior is proposed in paper of Shieh *et al*. [18]. Hosseinnezhad *et al*. [4] proposed A new method called Species-based Quantum Particle Swarm Optimization and applied in the power systems. Mahmoodabadi *et al*. [8] introduced a new optimization method based on the combination of PSO and two novel operators in order to increase the exploration capability of the PSO algorithm. In this paper, the developing Particle Swarm Optimization based on Minimax is proposed to optimize parameters of empirical virtually augmented formula, and in this way the virtually expanded samples will be more convincing and reliable.

To sum up, the Bootstrap method for reliability evaluation of small and extremely small samples was studied both at home and abroad, and applied in the fields of aircraft, missile and heavy machinery. In the aspect of intelligent algorithm optimization, PSO research emerges one after another and occupies a very important position. But no one has combined the two. After the sample size was expanded through virtual expansion, Bootstrap method was used for evaluation, and developing PSO proposed in the paper was used for parameter optimization to make the results more convincing, which is the significance of this paper.

2. The basis for Virtually Expanded Sample Method

Two basic conditions have to be met in order to keep the difference between random characteristic of new sample by virtually expanded and random characteristic of original sample within the engineering allowance, which are as follows:

1. The average of virtually expanded samples should be equal to the average of the original samples.
2. The standard deviation of virtually expanded samples should be equal to the standard deviation of the original samples.

On this basis, single sample of n=1 can be expanded to n=13. We do the following work. Assume that the distribution form of test estimation is normal distribution. To make the samples obtained by virtually expanded more reasonable, empirical formula of original virtually expanded samples is expressed as according to pertinent literature [12, 13]

$$x_n = x_0 \pm [a \cdot (i-1)^b + c]\sigma \tag{1}$$

where x_n is expanded samples value, n=1, 2, 3,⋯, 13. x_0 is original sample, σ is standard deviation. Because the standard deviation of single sample could not be calculated, the empirical standard deviation value is determined as 0.17, namely, σ=0.17. a and b are control coefficients to describe the dispersion characteristics of virtually augmented control points, which are different when the numbers of augmentation differ. c is a constant factor in order to satisfy the limiting conditions of the theoretical basis.

First of all, reserve the original sample x_0, and then 13 samples are expanded. The expanded samples represented by coefficients according to Equation (1) are as follows:

$$\begin{aligned}
x_1 &= x_0 - [a \cdot 5^b + c]\sigma,\\
x_2 &= x_0 - [a \cdot 4^b + c]\sigma,\\
x_3 &= x_0 - [a \cdot 3^b + c]\sigma,\\
x_4 &= x_0 - [a \cdot 2^b + c]\sigma,\\
x_5 &= x_0 - [a + c]\sigma,\\
x_6 &= x_0 - c \cdot \sigma,\\
x_7 &= x_0,\\
x_8 &= x_0 + c \cdot \sigma,\\
x_9 &= x_0 + [a + c]\sigma,\\
x_{10} &= x_0 + [a \cdot 2^b + c]\sigma,\\
x_{11} &= x_0 + [a \cdot 3^b + c]\sigma,\\
x_{12} &= x_0 + [a \cdot 4^b + c]\sigma,\\
x_{13} &= x_0 + [a \cdot 5^b + c]\sigma.
\end{aligned} \tag{2}$$

Here $\frac{1}{13}\sum_{n=1}^{13} x_n = x_0$ meets the first requirement of the virtually augmentation theory. Plus the virtually expanded samples x_1, x_2, …, x_{13}, into $\frac{1}{13-1}\sum_{n=1}^{13}(x_n - x_0)^2 = \sigma^2$. The result can be written as follows:

$$\frac{1}{12}[2 \cdot c^2 + 2(a+c)^2 + 2(a \cdot 2^b + c)^2 + \ldots + 2(a \cdot 5^b + c)^2] = 1 \tag{3}$$

After arrangement, we get

$$6c^2 + 2ac\sum_{n=1}^{5} n^b + a^2\sum_{n=1}^{5} n^{2b} = 6 \tag{4}$$

According to the Equation (4), the parameter c is obtained as follows:

$$c = \frac{-a\sum_{n=1}^{5} n^b \pm \sqrt{a^2(\sum_{n=1}^{5} n^b)^2 - 6(a^2\sum_{n=1}^{5} n^{2b} - 6)}}{6} \tag{5}$$

Combining the Equations (2) and (5), the samples expanded from n=1 to n=13 are obtained.

3. Bootstrap Method

3.1. Empirical Distribution Function

Arrange $x_{(1)}$, $x_{(2)}$, …, $x_{(n)}$, that represent their capacity and renumber them from small to large, namely,

$$x_{(1)} \le x_{(2)} \le \cdots \le x_{(n)} \tag{6}$$

Empirical Cumulative Distribution Function is expressed as follows:

$$F_n(x) = \begin{cases} 0 & x < x_{(1)}, \\ \dfrac{k}{n} & x_{(k)} < x < x_{(k+1)}, \\ 1 & x > x_{(n)}. \end{cases} \tag{7}$$

According to Gerry Brunei theorem, for any real number x, $F_n(x)$ converges to distribution function $F(x)$ with probability 1 when n approaches infinity. The expression form is as follows:

$$P\{\lim_{n\to\infty} \sup_{-\infty<x<\infty} |F_n(x) - F(x)| = 0\} = 1 \tag{8}$$

Therefore, for any real number x, when n is large enough, there are minute difference between observed value $F_n(x)$ and overall distribution function $F(x)$. And $F(x)$ can be replaced by $F_n(x)$.

3.2. Bootstrap confidence interval of mean value μ obtained by bootstrap-t

Bootstrap method could generate more samples based on existed samples, if the simulated samples comply with Bootstrap subsamples of Empirical Cumulative Distribution Function $F_n(x)$. According to the characteristics of Empirical Cumulative Distribution Function, the main steps generated Bootstrap subsamples are briefed as follows:

1. Generate the number m of uniform distribution at section [0, 1].
2. Ensure $l= m(n-1)$ and $c=[l]+1$, where $[l]$ represents that l is always rounded down.
3. x_F is a random sample point obtained by $x_F = x_{(n)} + (l-c+1)(x_{(n+1)} - x_{(n)})$.
4. $X_1 = \{x_{F_1}^{(1)}, x_{F_2}^{(1)}, \cdots x_{F_n}^{(1)}\}$.

The first Bootstrap sample, can be obtained through n times repetitions.

In this process, $X=(X_1, X_2, \ldots, X_n\ldots)$ is generated from Bootstrap samples which are obtained through manifold cycles. $x=(x_1, x_2,\ldots, x_n)$ is initial sample value. Random variables $X_1, X_2,\ldots, X_n\ldots$ are mutually independent and follow the same distribution. According to Central limit Theorem of Independent identical distribution, we get the expression as follows:

$$E(X_k)=\mu,\ D(X_k)=\sigma^2>0\ (k=1,2,\cdots) \tag{9}$$

Then the distribution function $F_n(x)$ of standardized variable of the sum of random variables $\sum_{k=1}^{n} X_k$, namely,

$$F_n(x) = \frac{\sum_{k=1}^{n} X_k - E(\sum_{k=1}^{n} X_k)}{\sqrt{D(\sum_{k=1}^{n} X_k)}} = \frac{\sum_{k=1}^{n} X_k - n\mu}{\sqrt{n}\sigma} \tag{10}$$

meets

$$\lim_{n\to\infty} F_n(x) = \lim_{n\to\infty} P\left\{\frac{\sum_{k=1}^{n} X_k - n\mu}{\sqrt{n}\sigma} \le x\right\} \tag{11}$$
$$= \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = \Phi(x)$$

That demonstrates when n is large enough, $\sum_{k=1}^{n} X_k$, the sum of random variables, follows normal distribution approximately, which is expressed as

$$\frac{\sum_{k=1}^{n} X_k - n\mu}{\sqrt{n}\mu} = \frac{\frac{1}{n}\sum_{k=1}^{n} X_k - \mu}{\sigma/\sqrt{n}} = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \tag{12}$$

It can be got that $\bar{X}$ follows $\bar{X} \sim N(\mu, \frac{\sigma^2}{N})$. Assume that $\bar{X}^*$ represents the corresponding sample values of Bootstrap respectively, and μ is replaced by $\bar{X}^*$, so we get

$$P\left\{\Phi^{(-1)}(\alpha) < \frac{\bar{X} - \bar{X}^*}{\sigma/\sqrt{n}}\right\} = 1-\alpha \tag{13}$$

Where $\Phi^{(-1)}$ is inverse of standard normal distribution, which means that following formula is true, namely,

$$P\left\{\bar{X}^* + \Phi^{(-1)}(\alpha)\frac{\sigma}{\sqrt{n}} < \bar{X}\right\} = 1-\alpha \tag{14}$$

When failure probability α is less than 50%, $\Phi^{(-1)}(\alpha)$ is negative. As mentioned above, lower confidence limit is as follows:

$$\bar{X}^* + \Phi^{(-1)}(\alpha)\frac{\sigma}{\sqrt{n}} \tag{15}$$

4. Equations Optimization of Empirical Equation Parameters

4.1. The Establishment of Objective Function And Constraint Conditions

The samples generated by virtual-augmentation function expanded from n=1 to n=13 are connected with parameters a and b, and a and b are control coefficients to describe dispersion characteristics of virtually augmented point. The values of a and b differ when the number of augmentation are different. Therefore, to generate Bootstrap samples which are more aligned with the situation expanded from n=1 to n=13, it is necessary to optimize the values of parameters a and b.

Optimization objective function is as follows:

$$f(a,b) = \left| x_0 - \Phi^{(-1)}(1-\alpha)\cdot 0.17 - (\bar{X}^* + \Phi^{(-1)}(\alpha)\frac{\sigma}{\sqrt{n}}) \right| \tag{16}$$

Where $\Phi^{(-1)}$ is the inverse function of cumulative distribution function of standard normal distribution.

The ultimate goal of optimization is the convergence of semi-empirical evaluation method and Bootstrap method. In this way, the optimum values of a and b are obtained, and experience Equation of expansion from n=1 to n=13 can be more consistent with the actual situation.

The constraint function is as follows:

$$g(a,b) = \begin{cases} g_1(a,b) = a^2(\sum_{i=1}^{5} i^b)^2 - 6(a^2\sum_{i=1}^{5} i^{2b} - 6) \ge 0, \\ g_2(a,b) = a \ge 0, \\ g_3(a,b) = b \ge 0, \end{cases} \tag{17}$$

Where the values of a and b are not negative.

4.2. Developing Particle Swarm Optimization Based on Minimax

4.2.1. Minimax Method

There are many methods to solve non-constrained optimization problems, so it is natural to try to transform constrained problems to non-constrained problems, which can be realized by Minimax.

The definition of Minimax is express as follows:

$$\begin{aligned} &\text{constrained problems } \min F(x)) \\ &\text{s.t. } g_i(x) \ge 0, i = 1,\cdots,m \end{aligned} \tag{18}$$

Non-constrained problems are as follows after conversion:

$$\begin{aligned} &\min f(x), \\ &f(x) = \max_{1 \le i \le m} f_i(x) \\ &F(x) = f(x) + \alpha_i g_i(x), \alpha_i > 0, 2 \le i \le m \end{aligned} \tag{19}$$

It can be proved when a_i is large enough, above constrained problems are equal to non-constrained problems. This strategy provides the large objective function for iteration points which tries to break the constraint conditions through solving non-constrained optimization problems. That forces minimum points of non-constrained optimization problems to be infinitely close to the feasible region, or keep moving within the feasible region until iteration points converge to the minimum point of the original problem.

4.2.2. Developing Particle Swarm Optimization

As a new type of stochastic evolutionary algorithm, Particle Swarm Optimization has been successfully applied in integer programming, neural network and electric power system due to ease of implementation, global search, fast convergent rate and few differences which needs adjustment. As one of heuristic random search methods, PSO evaluates systems with fitness functions and does random search according to fitness. Compared with traditional optimization methods, such as coordinate exchange method and steepest descent method, PSO is not easily trapped in local optimal solution. Meanwhile, during the process of search evaluation, PSO can decide search according to its own speed combining with memory function, then groups of particles are converged to the optimum quickly.

PSO is described as follows:

$$PSO = (N_{popu}, K_{iter}, V, P, F_{fit}) \tag{20}$$

Where N_{popu} is population size, $K_{iter}$is evolving algebra, V and P represent particle velocity space and location and location space respectively, F_{fit} is fitness.

Assume that there are m units in the population of D-dimensional space, the ith unit can be expressed as

$$x_i = (x_{i1}, x_{i2}, \cdots, x_{id}), i = 1, \cdots, m \tag{21}$$

The best historical space it went through is written as

$$p_i = (p_{i1}, p_{i2}, \cdots, p_{id}), i = 1, \cdots, m \tag{22}$$

The best space of all units belonging to the population is expressed as

$$p_g = (g_1, g_2, \cdots, g_d), i = 1, \cdots, m \tag{23}$$

For the *ith* particle of k generation, iterative Equations of particle swarm optimization algorithm are as follows:

$$\begin{aligned} &v_i^{k+1} = \omega v_i^k + c_1 r_1 (p_i - x_i^k) + c_2 r_2 (P_g - x_i^k) \\ &x_i^{k+1} = x_i^k + v_i^{k+1}, i = 1, \cdots, m \end{aligned} \tag{24}$$

Where r_1 and r_2 are random coefficients between [0, 1], ω is inertia weight, c_1 and c_2 are accelerating weight. Particle velocity v_i is restricted by maximum velocity, that is to say, $v_i > v_{\max}$, which can prevent the extreme disorder phenomenon. The main steps of standard particle group algorithm are briefly as the following:

- *Step* 1: Set parameters and initialize population.
- *Step* 2: Calculate the fitness value of each particle.
- *Step* 3: Compare adaptive value of every particle with the best historical position p_i. If there is the better, reset p_i;.
- *Step* 4: Compare adaptive value of every particle with the best historical position p_g of overall situation. If there is the better, reset p_g.
- *Step* 5: Update particles according to the evolution equation of stochastic particle swarm algorithm. Then return to step 2, and continue particle swarm iteration until the given number of iterations in advance is reached.

The inertia weight ω is used to control the influence degree of historical speed to current particle velocity. If ω is large, the capacity of algorithm to search for new area can be increased, but too large ω may lead to particle swarm explosion. And if ω is small, the capacity of algorithm to search for present area can be increased. Appropriate ω can keep balance between global searching ability and local search ability. Numerical experiments show that choosing the larger value at initialization stage and gradually reducing the value can obtain more accurate result.

In order to overcome the slow convergence of PSO algorithm in late period and avoid the calculation and storage of first derivative, global convergence of particle group algorithm, rapid convergence and high precision of Minimax method are combined to propose a new evolutionary algorithm. The process is written as follows:

- Search the global best point p_g using the stochastic particle swarm algorithm.
- Regard the final obtaining point as the initial point to search out a new point P_g' by coordinate rotation method.
- Replace p_g with P_g' to continue the particle swarm evolution of next generation. The validity and rationality of this algorithm have been proved by numerical experiments.

4.2.3. Test and Validation of Developing Particle Swarm Algorithm Based on Minimax

In order to demonstrate the effectiveness of the algorithm for nonlinear constraint problem, test functions are applied.

Question 1:

$$\begin{aligned} &\min f(x) = (x_1 - 2)^2 + (x_2 - 1)^2 \\ &x_1 = 2x_2 - 1 \\ &s.t.\ \frac{x_1^2}{4} + x_2^2 - 1 \le 0 \end{aligned} \quad (25)$$

Question 2:

$$\begin{aligned} &\min f(x) = x_1^2 + 4x_2^2 \\ &x_1 - x_2 - 1 \le 0 \\ &s.t.\ x_2 - 1 \le 0 \\ &-x_1 - x_2 + 1 \le 0 \end{aligned} \quad (26)$$

Results of testing functions can be seen in Table 1.

Table 1. Comparison of test function.

function	$f_1(x)$	$f_2(x)$
$f(x^*)$	1.39343	0.8
Best	1.39347	0.800000001
Average	1.3935	0.8000061
deviation	0.00004	0.000000001

4.3. The Optimization of Objective Function By Developing Particle Swarm Algorithm Based on Minimax

Based on the objective function (25) with constraint conditions (26), the objective function developing particle swarm optimization based on Minimax can be obtained as follows:

$$\begin{aligned} F(a,b) &= f(a,b) - \alpha_1 g_1(x) - \alpha_2 g_2(x) - \alpha_3 g_3(x) \\ &= \left| x_0 - \Phi^{(-1)}(1-\alpha)\cdot 0.17 - (\bar{X}^* + \Phi^{(-1)}(\alpha)\frac{\sigma}{\sqrt{n}}) \right| \\ &-\alpha_1\left(a^2(\sum_{i=1}^{5} i^b)^2 - 6(a^2\sum_{i=1}^{5} i^{2b} - 6)\right) - \alpha_2\cdot a - \alpha_3\cdot b \end{aligned} \quad (27)$$

Developing particle swarm algorithm is applied to optimize the objective function. Due to random factors of Bootstrap-t method and PSO algorithm, shock may be generated during the process of solving the optimal value. So it need to be optimized for many times to obtain optimal value and corresponding values of *a* and *b*.

5. Case

Fatigue life testing of a structure takes into account the test conditions including K_{py}=2.5, R=0, S_a=S_m=107(*Mpa*). Testing result of a specimen is N =159600, and logarithmic life standard deviation is 0.17 according to test experience. To ensure reliability reach 90%, reliability evaluation is applied to this structure. Because of single simple, the result can be expressed as: $T_0 = \lg N = 5.2030$.

To verify the superiority of this algorithm, Genetic Algorithm and PSO based on Minimax are applied simultaneously. Ten sets of data are obtained, which can be seen in Table 2.

Table 2. Comparison table of Genetic algorithm and Optimization Based on developing PSO based on Minimax.

Genetic algorithm			Developing PSO based on Minimax		
a	*b*	*F(a,b)*	*a*	*b*	*F(a,b)*
0.653	0.55	0.09124	0.607	0.458	0.09070
0.518	0.527	0.09162	0.467	0.654	0.09011
0.651	0.58	0.09062	0.352	0.813	0.09073
0.582	0.625	0.09070	0.568	0.666	0.09019
0.726	0.521	0.09104	0.479	0.704	0.08999
0.611	0.572	0.09094	0.58	0.656	0.09030
0.644	0.561	0.09144	0.484	0.744	0.09022
0.626	0.574	0.09071	0.697	0.538	0.09015
0.52	0.605	0.09089	0.664	0.424	0.09049
0.548	0.583	0.09137	0.469	0.656	0.09027

The results of the objective function obtained by the optimization of two algorithms can be seen in Figure 1.

By the comparison of the optimal value of objective function by two methods, means and variances of ten sets of data can be obtained (Table 3).

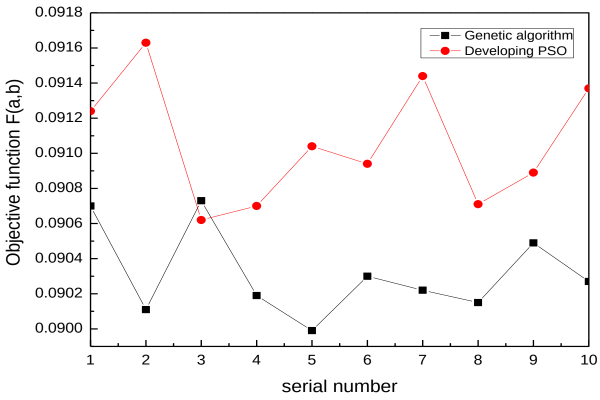

Figure 1. Comparison curves of Genetic algorithm and Optimization Based on developing PSO based on Minimax.

Table 3. The mean and variance of the optimal value calculated by the two methods.

	Genetic algorithm	developing PSO based on Minimax
The mean of the objective function	0.09106	3.46×10-4
The variance of the objective function	0.09031	2.48×10-4

From the Table 3, it can be observed that mean value of *F*(*a*,*b*) obtained by developing PSO based on Minimax is 0.83% less than genetic algorithm, and variance is also 39.52% less. When the objective function gets the minimum value, it means that variances of single sample virtually expanded method and semi-empirical evaluation method are the least. And in the process of calculation, the minimum variance between them is 0.08999. At this time, there are *a*=0.479 and *b*=0.704.

When $a = 0.479$ and $b = 0.704$ are achieved, variances of Bootstrap method and semi-empirical evaluation method are the least. The lower confidence limit of semi-empirical evaluation method is 5.0764, and Bootstrap method is 4.9864. Therefore, the lower confidence limit frequency of Bootstrap method is $10^{4.9852} = 9.6917 \times 10^4$.

6. Conclusions

In principle, Bootstrap method is more dispersed. Therefore, virtual-augmented sample method applied in this paper is more reasonable than semi-empirical method. By combing the global convergence of PSO with rapid convergence and high precision of Minimax method, developing PSO method which is based on Minimax was proposed. Under the premise that calculation and storage are needless, the lower convergence speed of PSO algorithm in late period was overcame. Parameters *a* and *b* in the process of sample virtual-augmentation were optimized with better speed, higher precision and smaller fluctuation. And in this way, estimated results were more in line with the practice and adjustability of practical engineering. The evaluation of single sample test had been achieved eventually.

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Xiaoxia Zhao is a PhD student in School of Mechanical Engineering, Taiyuan University of Science and Technology, Shanxi, China. She received her master degree from Taiyuan University of Science and Technology in 2015, Shanxi, China. Her research interests include intelligence algorithms, continuous conveyor, bulk material.

Wenjun Meng is a professor in Faculty of Mechanical Engineering, Taiyuan University of Science and Technology, Shanxi, China. At the same time, he is a vice-president in Shanxi Institute of Energy, Shanxi, China. He received his doctoral degree from Beihang University in 2005, Beijing, China. His research interests include intelligence algorithms, continuous conveyor, bulk material and artificial intelligence.

Jinhu Su is a PhD student in Faculty of Mechanical Engineering, Taiyuan University of Science and Technology, Shanxi, China. He received his master degree from Taiyuan University of Science and Technology in 2011, Shanxi, China. His research interests include intelligence algorithms, continuous conveyor, bulk material.

Yuxuan Chen is an engineer in North China Municipal Engineering Design & Research Institute Co. Ltd., Tianjin, China. She received her master degree from Taiyuan University of Science and Technology in 2015, Shanxi, China. Her research interests include intelligence algorithms, lifting and transportation machinery, bulk material.