Design and Analysis of Array Weighted Wideband Antenna using FRFT

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Abstract: *The beamwidth of a linear array depends on number of elements in the array and frequency of the input signal. The main requirement of wideband beamformer is, the main beam pattern should be constant even there is a change in input signal frequency. Various methods were proposed in literature, one method is called elemental lowpass filtering designed by using Finite Impulse Response (FIR) digital filters. In this paper, the elemental lowpass filtering method was implemented using Fractional Fourier Transform (FRFT) and performance analysis was carried out with array weighting.*

Keywords: *Antenna array, wideband antenna, FRFT, array weighting.*

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1. Introduction

Arrays of broadband signals have been applied in sonar, radio, radar, acoustic imaging etc., these are often difficult to design because of highly frequency dependent array properties. Figure 1 shows the directivity pattern of a simple 21 element linear array. The figure shows, the mainlobe width decreases with increase in frequency. This causes, some signals to be received with distorted spectra, and also frequency-dependent null locations impair the ability to cancel broadband interference.

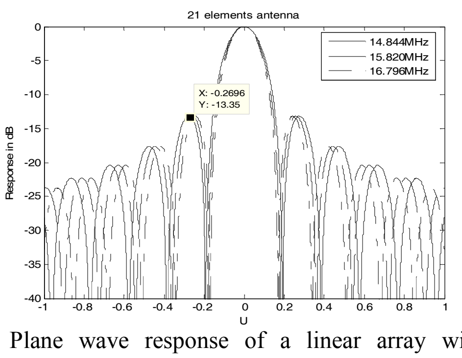


Figure 1. Plane wave response of a linear array with input frequency.

In the past, broadband beamformers have been studied extensively due to its applications [3, 4, 6]. It was apparent that, for a uniformly weighted linear array the largest sidelobes are down approximately 24 percent from peak value. The sidelobe levels can be further decreased by using non uniform spacing method. The presence of sidelobes means that the array is radiating energy in untended directions. In a multipath environment, the sidelobes can receive the same signal from multiple angles. This is the basis for fading experienced in communications [5]. The sidelobes can be suppressed by weighting, shading or windowing the array elements. Array element weighting has numerous applications in areas such as digital signal processing, radio astronomy, radar, sonar, and communications.

From Fractional Fourier Transform (FRFT) and related concepts [1, 2, 8], it has been seen that the properties and applications of Continuous Time Fourier Transforms (CTFT) is special case of FRFT. We have explained the usefulness of FRFT in the design of broadband beamformers with uniform spacing and advantages of it have been reported [7]. This paper gives the suitability and results of certain array weighting methods in the design of broad-band antenna array using FRFT. This paper is organized as: section 2 presents theory for beamforming, section 3 presents implementation of lowpass filter with FRFT, section 4 focused on some array weighting methods, section 5 gives the simulation results and section 6 represents conclusions respectively.

2. Beamforming Theory

For a linear array, the far field response for an input frequency ω and incident angle θ (measured relative to broadside) is given by [6]:

$$P(\theta, \omega) = \int_{-\infty}^{\infty} exp\left(-j\frac{\omega \sin\theta}{c}x\right)D(x,\omega)\,dx \tag{1}$$

where c and θ are the propagation speed and angle of impinging signal and $D(x,\omega)$ the frequency response with respect to the angular frequency ω and location x. Obviously, in general $p(\omega,\theta)$ is a function of both ω and θ, while for a frequency invariant beamformer, we require that the beam pattern $p(\omega,\theta)$ be independent of ω. For a weighted linear array of $2N+1$ equally spaced unidirectional elements, the far field response can be represented from the above equation 1:

$$P(\theta,\omega)=\sum_{n=-N}^{N} exp\left(-j\frac{\omega \sin\theta}{c}x\right)D(x,\omega)$$
$$=\sum_{n=-N}^{N} exp\left(-jn\omega\tau_0 \sin\theta\right)D(x,\omega) \quad (2)$$

where, $\tau_0=d/c$ is the interelement spacing divided by the speed of wave and $D(x,\omega)$ is the response of the filter connected to antenna element at x. The inter-null beamwidth of a uniformly excited array is given by [4]:

$$\theta_{BW}=2\sin^{-1}\left(\frac{2\pi}{M\omega\tau_0}\right)\approx\frac{4\pi}{M\omega\tau_0} \quad (3)$$

where, $M=2N+1$. This expression clearly indicates that the beamwidth is inversely proportional to frequency (or proportional to normalized frequency). It also implies that an increase in either the number of elements or interelement spacing results in a decrease in the beamwidth as well.

Figure 2 presents the transmission signal of signal from each antenna element and is passed through lowpass filter. The weighted sum of lowpass filters are combined at a linear combiner [3]. Here, we have used FRFT to design lowpass filter.

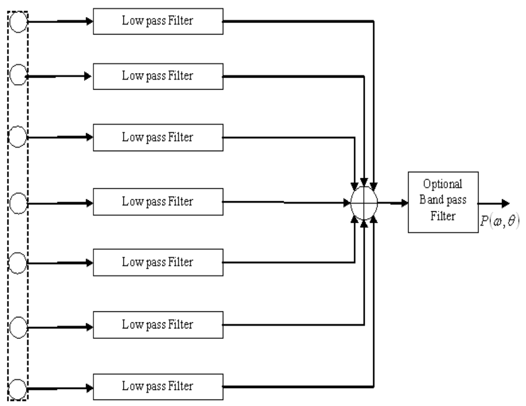


Figure 2. Filter-and-sum structure.

3. Implementation of Low Pass Filter using FRFT

The CTFT, which is defined by the following pair [8]:

$$F(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\,exp(-j\omega t)\,dt \leftrightarrow$$
$$f(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(\omega)\,exp(j\omega t)\,d\omega \quad (4)$$

The CTFT reflects to the assumption that the signal of interest has stationary frequency content. However, signal representations using intermediate "angularly coupled axes" hold some promise for analyzing signals with time-frequency coupling, e.g., linear-frequency modulation. Angular transform of the CTFT, which is called as the Angular Fourier Transforms (AFT) or FRFT, which is controlled by a single continuous angular parameter α. So, FRFT can be represented as the rotation of signal in time-frequency plane [8] as shown in Figure 3.

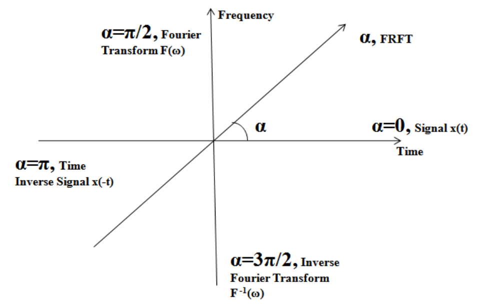


Figure 3. Rotation of signal in time frequency plane with an angle α.

The generalization of the CTFT is obtained if we consider a rotation through an arbitrary angle α in the $(t,\ \omega)$ plane. Thus, the FRFT of a signal $f(t)$, can be expressed as [7]:

$$F^a[f(t_a)]=\int_{-\infty}^{\infty} K_a(t_a,t)f(t)dt$$
$$K_a(t_a,t)=K_\varphi exp\left[j\pi\left(t_a^2\cot\varphi-2t_a t\,cosec\varphi+t^2\cot\varphi\right)\right] \quad (5)$$

$$K_\phi=\exp\left[-j\left(\pi\,\mathrm{sgn}(\phi)/4-\phi/2\right)\right]/\left[\sin\phi\right]^{0.5} \quad (6)$$

where $\phi=a\pi/2$.

The kernel function $K_a(t_a,t)$ has the following spectral expansion:

$$K_a(t_a,t)=\sum_{k=0}^{\infty}\psi_k(t_a)\exp\left(-j\frac{\pi}{2}ka\right)\psi_k(t) \quad (7)$$

where $\psi_k(t)$ denotes kth Hermite-Gaussian function, and t_a denotes the variable in the ath-order Fractional Fourier Domain. The kth order Hermite-Gaussian function is defined as ($k=0,1,2,\ldots$):

$$\psi_k(t)=\frac{2^{1/4}}{\sqrt{2^k k!}}H_k\left(\sqrt{2\pi}\,t\right)\exp\left(-\pi t^2\right) \quad (8)$$

where H_k denotes kth order Hermite polynomial having k real zeros. In the equation 7, $\exp\left(-j\frac{\pi}{2}ka\right)$ represents the ath power of the eigenvalues. When $a=1$, the FRFT reduces to the ordinary fourier transform**,** where t_1 denotes the frequency-domain variable. Here, we have adopted the design low pass filter using FRFT based on procedure of tunable Finite Impulse Response (FIR) filters. A FIR digital filter operation is a linear convolution of the finite duration impulse response with the input signal sequence $x(n)$. The impulse response $h(n)$, of kaiser window is given as [9]:

$$h(n)=h_d(n)w(n) \quad (9)$$

where $h_d(n)$ is the desired or ideal impulse response, and $w(n)$ is the kaiser window sequence. Since multiplication in the time domain corresponds to convolution in the frequency domain which can be expressed as a complex convolution operation given as:

$$H(\omega) = \frac{1}{2\pi}\int_{-\pi}^{\pi} W(\lambda) H_d(\omega-\lambda)\, d\lambda \tag{10}$$

where, $H(\omega)$ is the frequency response of filter $H_d(\omega)$, is desired or ideal frequency response of lowpass filter, and $W(\omega)$ is frequency response of kaiser window. From the above equation, the transition bandwidth of is proportional to mainlobe width of kaiser window $W(\omega)$. Figure 4 shows the variation of mainlobe width with order of FRFT. It is observed that as the FRFT order is reduced the main lobe width of FRFT kaiser window shrinks. This feature has been used here to tune the transition bandwidth of lowpass filter.

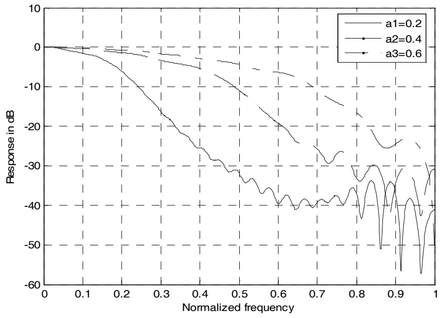


Figure 4. Variation in kaiser window response with order of FRFT.

4. Array Weighting Methods

There are a vast number of possible window functions available that can provide weights for use with linear arrays. Some of the more common window functions are considered for analysis of wideband antenna.

4.1. Binomial Weights

Binomial weights will create an array factor with no sidelobes, provided that the element spacing $d \le \lambda/2$. The binomial weights are chosen from the rows of Pascal's triangle. The first five rows are shown in Table 1.

Table 1. Pascal's triangle.

N = 1					1				
N = 2				1		1			
N = 3			1		2		1		
N = 4		1		3		3		1	
N = 5	1		4		6		4		1

4.2. Gaussian Weights

The gaussian weights are defined by:

$$w(k+1) = e^{-\frac{1}{2}\left(\alpha\frac{k-N/2}{N/2}\right)^2} \tag{11}$$

where $k=0, 1,\ldots, N$ and $\alpha \ge 2$.

4.3. Hamming Weights

The hamming weights are defined by:

$$w(k+1) = 0.54 - 0.46\cos(2\pi k/(N-1)) \tag{12}$$

where $k=0, 1,\ldots, N-1$.

4.4. Kaiser-Bessel Weights

The kaiser-bessel weights are defined by:

$$w(k) = \frac{I_0\left[\pi\alpha\sqrt{1-\left(\frac{k}{\lambda/2}\right)^2}\right]}{I_0[\pi\alpha]} \tag{13}$$

where $k=0, 1,\ldots, N-1$ and $\alpha>1$.

4.5. Blackman-Harris Weights

The blackman-harris weights for -92dB sidelobe level are defined by:

$$\begin{aligned} w(k+1) &= 0.35875 - 0.48829\cos\left[\frac{2\pi k}{N-1}\right] \\ &+ 0.14128\cos\left[\frac{4\pi k}{N-1}\right] . - 0.01168\cos\left[\frac{6\pi k}{N-1}\right] \end{aligned} \tag{14}$$

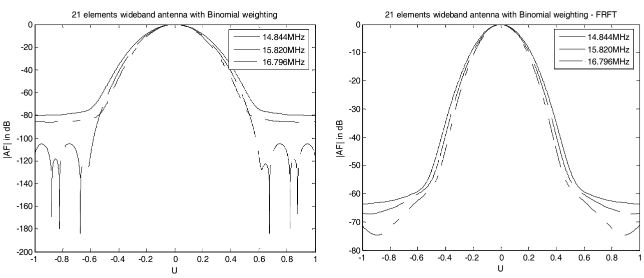
4.6. Nuttall Weights

The Nuttall weights are defined by:

$$\begin{aligned} w(k+1) &= 0.35576 - 0.48739\cos\left[\frac{2\pi k}{N-1}\right] \\ &+ 0.14423\cos\left[\frac{4\pi k}{N-1}\right] . - 0.01260\cos\left[\frac{6\pi k}{N-1}\right] \end{aligned} \tag{15}$$

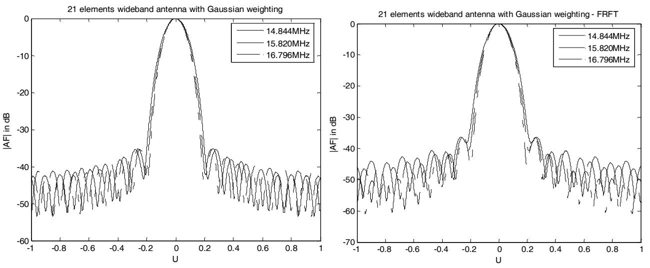
5. Simulation Results

The design results were evaluated using computer simulation, for frequencies 14.844MHz, 15.820MHz, and 16.796MHz, for 21 element antenna array with uniform spacing. Figures 5, 6, 7, 8, 9 and 10 show the antenna pattern and the frequency characteristics of the wideband antenna with FIR Low Pass filter and FRFT Low Pass Filters for Binomial, Gaussian, Hamming, Kaiser-Bessel, Blackman-Harris and Nuttall weighting methods.



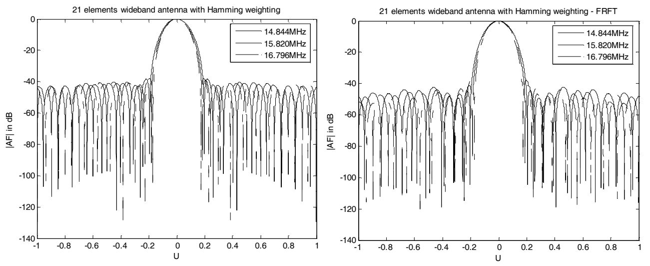
a) FIR low pass filter. b) FRFT low pass filter.

Figure 5. Response of 21-element wideband antenna array designed with binomial weights.



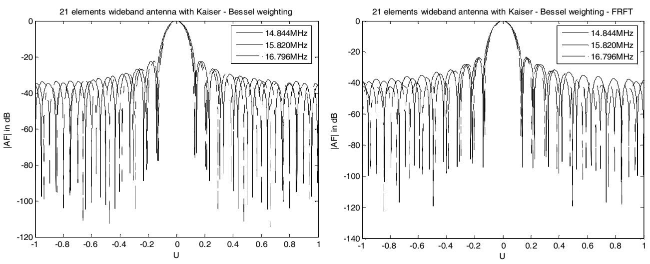
a) FIR low pass filter. b) FRFT low pass filter.

Figure 6. Response of 21-element wideband antenna array designed with Gaussian weights.



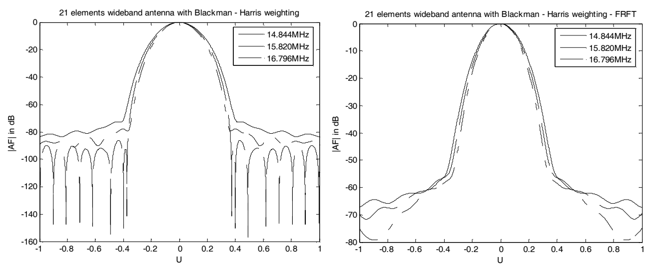
a) FIR low pass filter. b) FRFT low pass filter.

Figure 7. Response of 21-element wideband antenna array designed with Hamming weights and FRFT Low Pass Filter.



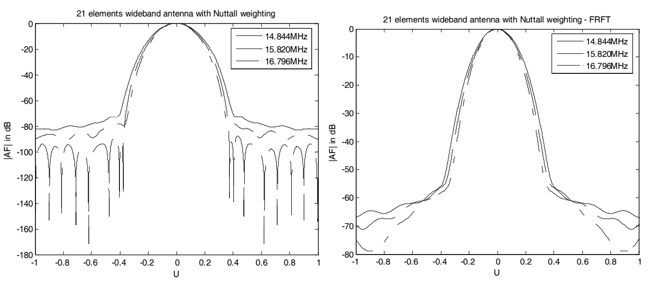
a) FIR low pass filter. b) FRFT low pass filter.

Figure 8. Response of 21-element wideband antenna array designed with Kaiser-Bessel weights.



a) FIR low pass filter. b) FRFT low pass filter.

Figure 9. Response of 21-element wideband antenna array designed with Blackman-Harris weights.



a) FIR low pass filter. b) FRFT low pass filter.

Figure 10. Response of 21-element wideband antenna array designed with Nuttall weights.

6. Conclusions

In this paper we have discussed about application of array weighting on the design of wideband antennas. From the Figures 5, 6, 7, 8, 9 and 10, it is evident that if the low pass filters are implemented by using FRFT, gives better invariant radiation patterns compared to normal FIR low pass filters. Figures 5, 9 and 10 shows that the Binomial weights, Blackman-Harris weights and Nuttall weights gives lower side lobe levels, more than 80dB with FIR filters and more than 60dB with FRFT filters. From Figure 8 it is evident that kaiser-bessel weights gives better directional characteristics.

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