A Hybrid Approach for Modeling Financial Time Series

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Abstract: *The problem we tackle concerns forecasting time series in financial markets. AutoRegressive Moving-Average (ARMA) methods and computational intelligence have also been used to tackle this problem. We propose a novel method for time series forecasting based on a hybrid combination of ARMA and Gene Expression Programming (GEP) induced models. Time series from financial domains often encapsulate different linear and non-linear patterns. ARMA models, although flexible, assume a linear form for the models. GEP evolves models adapting to the data without any restrictions with respect to the form of the model or its coefficients. Our approach benefits from the capability of ARMA to identify linear trends as well as GEP's ability to obtain models that capture nonlinear patterns from data. Investigations are performed on real data sets. They show a definite improvement in the accuracy of forecasts of the hybrid method over pure ARMA and GEP used separately. Experimental results are analyzed and discussed. Conclusions and some directions for further research end the paper.*

Keywords: *Financial time series, forecasting, ARMA, GEP, hybrid methodology.*

Received February 6, 2010; accepted October 24, 2010

1. Introduction

Financial time series modeling has been an active area of research for decades. The interest is two-fold. Firstly, time series models may shed light into the laws governing the behavior of the data generating process, providing a description of the variation of price over time. Secondly, good models may lead to accurate predictions of the future behavior, therefore, to profitable trading strategies [5, 20].

The classical economic theory of the Efficient Market Hypotheses (EMH) states that the market is efficient. Consequently, no profit can ever be made based on any kind of information. The main assumptions of the EMH include: all players on the market make rational decisions, they have access to all available information, and they have the same abilities to process and analyze this information. In its current form, the EMH is summarized by Lo [13], by the "three P's": prices, probabilities, and preferences.

Many critiques of this theory come from different angles, such as behavioral economists or psychologists. They state that decisions are not always rational, hence not considering the emotions of the agents in a market faults the entire promoted model of that market. Moreover, from a computational perspective, it is very clear that the data mining methods and computational resources are very diverse among market players, thus there may be ways to find a trading edge to beat the market and make profit.

Recently, a new theory has been proposed the Adaptive Market Hypotheses (AMH) [13]. It brings the principles of Darwin's evolution theory into the economic scenery. The AMH promotes the idea of an evolving market, where dynamism allows for brief periods when market efficiency can be surpassed, patterns appear and profit can be made by acting upon these patterns. Lo [13] states that the market follows an evolutionary model, driven by the basic principles of natural evolution-competition, adaptation and natural selection. The financial environment is continuously changing, while the individuals in this environment struggle to adapt.

There are numerous reports that combat the EMH even in its weak-form, using run tests or tests based on trading rules [17]. Our study is aligned to these results. We obtain accurate models of time series, based on past knowledge of stock price time series. The method used for this purpose relies on the use of a solid statistical technique combined with a novel evolutionary method. Our intention is to verify empirically whether the combination brings any improvement over the two traditional methods.

1.1. Related Work

There exists a multitude of methods to perform time series modeling, stemming from various areas like signal processing, dynamical systems, statistics, technical analysis, or artificial intelligence. The traditional methods come from the statistics literature, and include exponential smoothing, autoregressive, nonlinear threshold or autoregressive conditional heteroscedastic models. Modern methods rely on the use of Artificial Neural Networks (ANN) [10, 16] or algorithms that belong to the field of Evolutionary Computation (EC) [21]. Belov *et al*. [4] investigate the usefulness of stable models in the stock market. De-Gooijer [9] provide an extensive review of the development in this field in the last 25 years. They account for the importance of parsimony along with accuracy in modeling, a fact that impairs the use of nonlinear models, such as ANN.

The Box-Jenkins methodology is a very powerful and expressive branch of research. ARMA models are easily interpretable and their applications are diverse and successful [5]. Yet, the methodology comes with some limitations. It assumes the stationarity of the time series, the normality and independence of the residuals. In addition, these models lack the ability to identify complex non-linear traits in the data. Nonlinearities are often "at fault" for the interestingness and difficulty behind financial time series [13].

Heuristic approaches based on ANN or EC have been shown to obtain amazing results in time series modeling and forecasting. ANNs construct "black box" models of time series [10], useful for forecasting, useless for characterization of the system. They have good approximation capacities, but are often poor at generalization, due to overfitting [23]. Evolutionary algorithms appear in two contexts when time series are concerned. One of them is when genetic algorithms are used to enhance the performance of ANNs, by evolving the weights or the architecture of the network [24].

The other involves genetic programming (or its variants), either for evolving informational structures of the time series such as decision trees, or for evolving analytical functions that provide point forecasts of the time series, based on a number of past values. The genetic programming approach in [19] reported results that outperformed AutoRegressive (AR) and Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) models constructed for the same financial time series. In addition, the GP system described in [21] obtained good predictions when applied in a dynamic financial setting.

Gene Expression Programming (GEP) is a recently introduced evolutionary technique, targeted towards automatic induction of models for data [7, 8]. One of GEP's main strengths is the nonlinear modeling capability. As opposed to usual statistical techniques, GEP obtains model of no-predefined form, finding both the appropriate shape of the function and its coefficients through evolution. GEP was used to develop a system that obtains simple mathematical models of the temporal behavior of data in [14]. Good results were reported when combining statistical data analysis with GEP modeling for meteorological time series [3].

1.2. Motivation

Typically, time series modeling is an iterative process: several models of different classes are built, analyzed, and usually, the one that obtains the smallest errors is chosen. The downside is that error based criteria that characterize an entire model by a single number, leaving a lot of space for forecasting errors.

A common approach to obtain robustness of time series models is to build them using more than one technique. Moreover, in situations where time series reflect both linear and nonlinear traits, the best approach is to try and model them individually. Since there exists no universal method to obtain the best models with the best forecasts, attempts are constantly being made to find the most appropriate method, depending on the occasion.

The technique of combining several models for the same time series has been used intensely. Early reports come from Clemen [6], while good and more recent reviews are offered by Armstrong and De-Gooijer [2, 9]. Studies have shown the increase in prediction efficiency especially when combining. Liang [12] uses a hybrid method that combines in a serial manner seasonal AutoRegressive Integrated Moving-Average (ARIMA) modeling with a neural network.

The combined model reduces the prediction error in comparison to a pure neural network model. A similar approach is used in [15], where ARMA modeling is combined with a Support Vector Machine (SVM) in the analysis of some financial time series. Comparisons between the computational results of the base methods (ARMA and SVM) and those of the hybrid method indicate that the hybrid outperforms both traditional approaches. In [1], the authors provide empirical results that a combination of an ARIMA model and an ANN has improved forecasting accuracy over the plain ARIMA and ANN approaches.

In this paper, we propose a novel approach using a hybrid combination of ARMA and GEP. To the best of our knowledge, no approach like this was reported in the literature. The motivation of our study is to investigate to what extent two different time series modeling methods ARMA and GEP can complement each other in the analysis of financial time series. We decompose the time series into two components: one that captures linear behavior and one for the non linear one. We intend to take advantage of ARMA's capacity to model the linear component and then to derive a model for the residuals obtained from the ARMA modeling using GEP. This way, if it exists, the nonlinear behavior would be reflected in the residuals obtained after extracting the linear component and they would be captured by means of GEP modeling. The resulting combined model is expected to be more robust and fit more accurately.

2. Time Series Models

2.1. ARMA Models

A time series model for the observed data x_t is a specification of the joint distributions of a sequence of random variables X_t of which x_t is postulated to be a realization. Let X_t be a discrete process in time and let us consider the operators defined by:

$$B(X_t) = X_{t-1} \tag{1}$$

$$\Phi(B) = 1 - \varphi_1 B - ... - \varphi_p B^p, \ \varphi_p \neq 0 \tag{2}$$

$$\Theta(B) = 1 - \theta_1 B - ... - \theta_q B^q, \ \theta_q \neq 0 \tag{3}$$

X_t is said to be an $ARMA(p,q)$ process if $\Phi(B)X_t = \Theta(B)\varepsilon_t$, where the absolute values of the roots of Φ and Θ are greater than 1 and ε_t is a white noise. An $ARMA(p,q)$ process is called an $AR(p)$ process if $q = 0$. An $ARMA(p,q)$ process is called a $MA(q)$ process if $p = 0$ [5].

The Box-Jenkins methodology of finding the appropriate ARMA model for a time series contains three steps: the first is concerned with identifying the model type (i.e., its order), the second with identifying the suitable parameters, and the third consists in checking the model. When the time series is not stationary, some processing (differencing, transformation) may be performed in order to reach the desired statistical properties. In order to determine the process type, the autocorrelation properties of the series are investigated.

2.2. Gene Expression Programming

Evolutionary computation is a branch of artificial intelligence dedicated to meta heuristics that use ideas from biological evolution to solve optimization problems. Many techniques are part of this field (e.g., genetic algorithms, evolutionary strategies, genetic and evolutionary programming). The common trait to all these methods is that they are governed by the principles of natural evolution. Amongst them, the lead is the principle of natural selection: the individual that is best adapted to the environment has the greatest chances to survive, reproduce and therefore pass his genetic traits to the next generations. A population of candidate solutions is initialized and then goes through a process of selection, recombination in a loop, until some termination criterion is met.

Genetic programming appeared around the 90's with the purpose of achieving automatic programming [18]. Empowered by the good results of GP, many improved variants of the technique were proposed. Among them lies GEP, as being one of the most successful paradigms.

GEP is a flavour of GP that uses a novel representation that also takes advantage of some features of the classical GA. It works with a population of candidate solutions, in our case complex mathematical functions obtained as compositions of elementary functions, with variables and constants. Individuals are complex functions, free from any constraints regarding their form. GEP individuals are fixed size strings of symbols of the same length. Nonetheless, they encode non-linear expressions. An individual is composed of one or more genes of equal length; the number of genes is constant throughout the population over all generations and is given as a parameter of the algorithms, as is the gene size.

The symbols that may appear in a GEP chromosome are functions or terminals. By functions we understand mathematical functions, while the terminals are either constants or variables. The variables are the same as they would be for any other time series modeling method: they represent lagged values of the time series. The number of past values used by GEP is a parameter of the algorithm.

If we denote the values in the series by $(x_t)_{t\in\overline{1,n}}$, and the values estimated by the GEP model by $(\hat{x}_t)_{t\in\overline{1,n}}$, we are interested in finding a function f that predicts the values of a time series as accurately as possible:

$$\hat{x}_t = f\left(x_{t-1}, x_{t-2}, ..., x_{t-w}\right), \ t \leq n \tag{4}$$

The function has no predefined analytical form, nor specific coefficients- it is obtained entirely by evolving mathematical expressions in a GEP algorithm. The accuracy of a model for a series of n observations is measured in terms of Mean Squared Error (MSE):

$$MSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \hat{x}_i\right)^2} \tag{5}$$

Better models are those with smaller error. The fitness assigned to GEP individuals is this error, thus the algorithm works towards minimizing the fitness. For details on the inner workings of GEP see [7, 8].

3. The Hybrid ARMA-GEP Approach

ARMA and GEP have abilities to focus on different aspects of time series data. In this study, they are combined in order to provide more robust and accurate models of time series. To this end, the time series is modeled by a sum of a linear and a nonlinear component:

$$\hat{x}_t = \hat{a}_t + \hat{Z}_t \tag{6}$$

The first component will be determined by an ARMA model in order to catch the linear structure of data. The $\hat{a}_t$ symbolizes the approximation obtained by the linear model for the series value at the moment t. Then, a GEP model will be fit. In fact, the residuals of he ARMA model, $(x_t - \hat{a}_t)$ are the ones to be fit by GEP models. Residuals are very important in ARMA modeling, since the existence of nonlinear patterns in the residuals series compromise the ARMA model. Therefore, modeling the residuals in order to identify nonlinear patterns strengthens the ARMA models obtained, complementing it. This separate modeling helps to identify different patterns in the series.

The resulted models benefit from both ARMA's capacity to model the linear component and the complex nonlinear structures evolved by GEP. If it exists, the nonlinear behavior of the original time series is reflected in the residuals obtained after extracting the linear component and it would be captured by means of GEP modeling. Armstrong [2] details some conditions favoring combinations of forecasts resulted from different methods. He mentions that the use of combinations of models is recommended for time series when the most appropriate model is not known, or when it is important to avoid large errors. This combination has the features that recommend it as providing improvements with respect to the individual models.

4. Empirical Results

We are interested to establish whether the combination of ARMA modeling with GEP for financial time series brings improvement over the models obtained by ARMA or GEP alone.

4.1. Input Data

The experiments reported in this paper are performed on time series of monthly close stock prices* coming from both mature and emergent markets. This way we investigate the usefulness of modeling techniques employed in diverse settings, since market efficiency influences drastically the predictability of stock [13]. Four important indices were studied:

1. New York Stock Exchange (NYSE) - between 12.1965 and 07.2009 (526 values).
2. Bucharest Exchange Trading (BET) - between 10.2000 and 06.2009 (105 values).
3. Bursa Malaysia KLCI Index (KLSE) - between 12.1993 and 06.2009 (187 values).
4. Dow Jones Industrial Average (DJIA) index- between 05.1896 and 06.2009 (1354 values).

The first and the last indices come from mature financial markets. The second and third indices come from emerging markets (BET and KLSE). The last index is the oldest existent index, coming from the mature American stock market (DJIA). Expectations are that the BET and KLSE indices are easier to model than the first and last, given that the American stock market is expected to be more efficient than new markets (more players, more experienced, a greater transactions volume). Moreover, we expect the results for the first index to be influenced by the length of the time series, which is bigger than BET and KLSE series.

The settings for GEP parameters were done as recommended in [8] for symbolic regression problems. All experiments were performed with chromosomes up to 5 genes, with the number of symbols in the head set to 5. The function set used is $\{+,-,*,/,\sin\}$. All GEP models used up to 12 historical values. The best results were obtained for small window sizes (less than equal to 3). We include here the models for a window size of 2, because they are good in terms of error, and still of a manageable complexity. The models were chosen from the best solutions encountered in 50 independent runs (distinct random seeds on each run), with the condition that its residuals are independent and normally distributed.

4.2. Models

4.2.1. NYSE Series

The NYSE series is not gaussian and the normality can not be reached through variable transformations. Also, the series is correlated. After the mean subtractation, the model for the new series, is an AR(2):

$$X_t = 1.219\,X_{t-1} - 0.2215X_{t-2} + \varepsilon_t \tag{7}$$

where ε_t has the variance of 27250.3.

Since the residuals' variance was too big and the residuals are correlated and not normal, another model was determined for the series obtained after taking logarithms, and denoted by Y_t. The best model identified, applying the Akaike selection criterion, was:

$$Y_t = 0.998\,Y_{t-1} + Z_t \tag{8}$$

where the variance of the white noise Z_t was 0.002651. The residuals from equation 8 were modeled with GEP Figure 1.

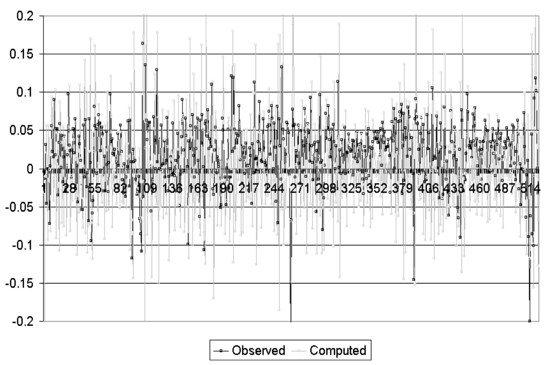

Figure 1. The GEP model of the residuals for the 1n(NYSE) series (obtained with a window size of 2).

* Data downloaded from http://www.stooq.com.

The analytical solution is:

$$\hat{Z}_t = \frac{Z_{t-1} - Z_{t-2}}{4 + 0.15 Z_{t-2}} + Z_{t-2}^2 + Z_{t-2} - Z_{t-1} \qquad (9)$$

The hybrid model resulted by combining the two methods is depicted in Figure 2. The corresponding residuals have a mean of -0.017 and a standard deviation of 0.08.

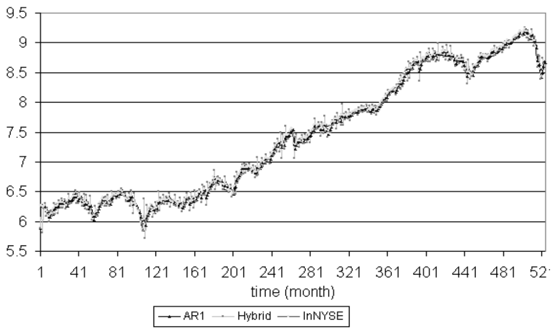

Figure 2. Models of the transformed 1n(NYSE) series.

4.2.2. BET Series

The Kolmogorov-Smirnov test of normality lead us to reject the normality hypothesis on BET series. This series presents autocorrelation (noted from the chart of ACF). The form of the partial ACF chart Figure 3 recommends an AR type model. The best model was chosen, based on Akaike criterion, after the mean extraction. It has the equation:

$$X_t = 0.983\, X_{t-1} + \varepsilon_t \qquad (10)$$

where ε_t was a white noise with the variance of $2.6E+07$.

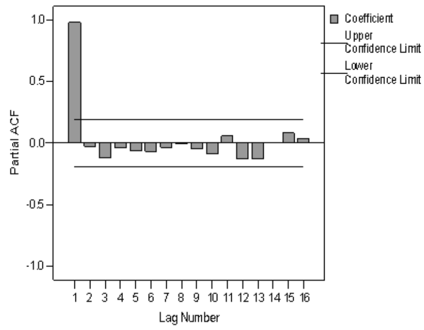

Figure 3. PACF of BET series.

Since the variance of the residuals in equation 10 is very high and a delay between the model and the observed data can be seen Figure 4, the data underwent a logarithmic transformation. The new series Y_t, is not normally distributed and the normality could not be reached through new transformations. After the mean subtraction, the best model determined is of ARMA (1, 1) type:

$$Y_t = 0.994 Y_{t-1} + Z_t + 0.248 Z_{t-1} \qquad (11)$$

where the white noise Z_t has a variance of 0.011.

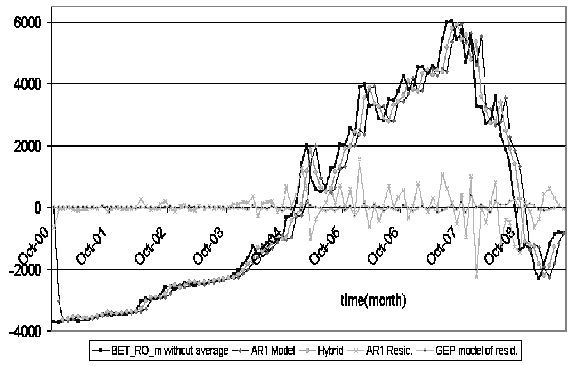

Figure 4. Models of BET after the extraction of the mean.

The GEP model of the residuals of equation 11 was obtained for a history window size of 2 and its analytical expression is:

$$\hat{Z}_t = -0.72 - Z_{t-2} \cdot Z_{t-1} + \frac{Z_{t-1}}{0.74} - 2.28 \cdot Z_{t-1}^3 \cdot Z_{t-2} \qquad (12)$$

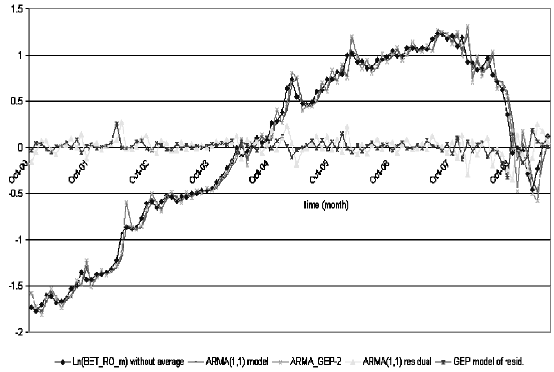

Figure 5. Models of 1n(BET)-the original series, the ARMA model, the hybrid model.

Although the nature of the residual series is observably complex, the model obtained captures the nonlinear pattern and, most of all, the trend of the data. The models of transformed series, denoted by 1n(BET), are plotted in Figure 5. The hybrid not only looks as a better fit for the original series, it also brings an improvement in the MSE of 6% for the prediction over the first 7 months of year 2009. The residual in (11) is also plotted in Figure 6. It obviously portrays a good fit for the nonlinear pattern shown by the Z_t.

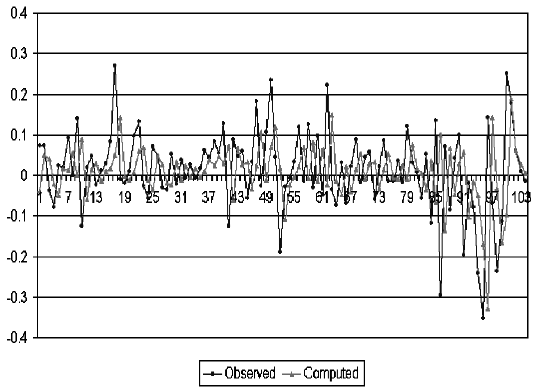

Figure 6. The ARMA residual and its values computed by the GEP model.

The advantage is that, while using only the ARMA model for forecasting, one would predict the next value to be the previous value in the series, the hybrid approach models also the residual part (the difference between the two), enabling the prediction to be more focused and closer to the real value. Hybridization thus improves the prediction, and provides further insight than only taking the most recent observation as a guide for the next prediction.

4.2.3. KLSE Series

The hypothesis that the series follows the normal law was accepted after performing Kolmogorov-Smirnov and Shapiro- Wilk tests. Applying the rank correlation test, the null hypothesis (randomness of time series) is rejected at the confidence level of 95%. No further transformation of the series is needed. The best model obtained for the raw series is an ARMA (1, 1):

$$X_t = 0.996 X_{t-1} + \varepsilon_t + 0.017 \varepsilon_{t-1} \quad (13)$$

where the variance of the residual ε_t is 3592.95. The residuals are normal, but correlated. The hybrid combination with the GEP model for ε_t succeeds in removing the delay (the ARMA model was visibly delayed). The improvement in MSE of the hybrid over the pure ARMA is only 1% when predicting for the year 2008 Figure 7.

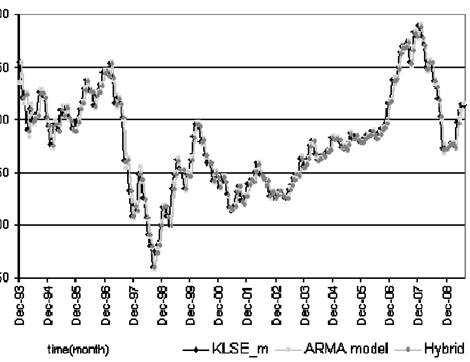

Figure 7. Models for the original KLSE series.

In order to obtain a better result, after a logarithmic transformation the series obtained Y_t, was also investigated. The hypothesis that it is normally distributed was rejected at the confidence level of 95%. Also, the new series has some outliers, which might represent some rare events that influenced the behaviour of the time series.

Since we are interested in finding a model that follows the general trend, the new series without the outliers was considered. The best model determined with respect to the AIC criterion is an ARMA (1, 1):

$$Y_t = 0.9997 Y_{t-1} + Z_t + 0.1817\, Z_{t-1} \quad (14)$$

where Z_t has the variance of 5470. The residuals Z_t are correlated and not normal. The model found by GEP for them captures the nonlinearities and predicts quite well:

$$\hat{Z}_t = -0.02 \cdot Z_{t-2} - 0.26 \frac{Z_{t-1}}{Z_{t-2}} \quad (15)$$

when combined with the ARMA model, the hybrid is a good fit of the original series Figure 8.

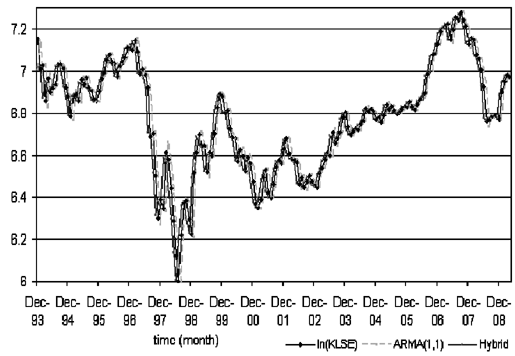

Figure 8. Models for 1n(KLSE).

4.2.4. DJIA Series

The series is not normally distributed, but the normality can be reached by a Box-Cox transformation with $\lambda = 1.38$. The values of ACF are strictly decreasing and the first value of the Partial ACF is outside the confidence interval, at 95% confidence level. After performing the rank correlation test, the null hypothesis of time series randomness was rejected at the confidence level of 95%. The series reveals a large number of outliers, as expected, because the time span for this series covered many important events that had direct impact on the economic environment, hence reflected in the evolution of the stock price.

Given the length of the series, there is a high chance that structural breaks affected the time series. Indeed, applying Buishard and Bayesian (Lee and Heghinian) methods, the hypothesis that there is no break in the time series is rejected at the confidence level of 95%. Still, since we are not interested in modeling the series in a piecewise model, we will ignore the breaks for the moment and concentrate ourselves on obtaining an overall fit model for the entire series. The best model encountered by the Box-Jenkins methodology was an AR (1):

$$X_t = 0.999 X_{t-1} + \varepsilon_t \quad (16)$$

where the residuals ε_t have the variance of 24596.3 and they are not normal, nor correlated. They are modeled further by GEP. A complex model is derived, that reflects the nonlinearities, yet doesn't explain all the amplitude. Combining the AR model with the GEP model, the MSE of the resulted hybrid model is not improved. Hence, we look for a better model. We apply a logarithmic transformation; the resulted series, is not normal, it is correlated, but has no outliers. Pettitt test leads us to accept the hypothesis that there is no break in the new time series. We further extract the mean, and model the resulted series Y_t, by a $MA(12)$ model:

$$\begin{aligned} Y_t = & Z_t + 2.988\, Z_{t-1} + 2.903\, Z_{t-2} + 2.588\, Z_{t-3} + \\ & + 2.721\, Z_{t-4} + 2.662\, Z_{t-5} + 2.502\, Z_{t-6} + \\ & + 2.592\, Z_{t-7} + 2.994\, Z_{t-8} + 3.103\, Z_{t-9} + \\ & + 3.011\, Z_{t-10} + 3.228\, Z_{t-11} + 3.254\, Z_{t-12} \end{aligned} \quad (17)$$

where residual Z_t is a white noise. Modeling Z_t by GEP the equation was obtained:

$$\hat{Z}_t = \frac{Z_{t-1} - Z_{t-2}}{-0.15 \cdot Z_{t-1} + 3.99} + Z_{t-2}^2 + Z_{t-1} - Z_{t-2} \quad (18)$$

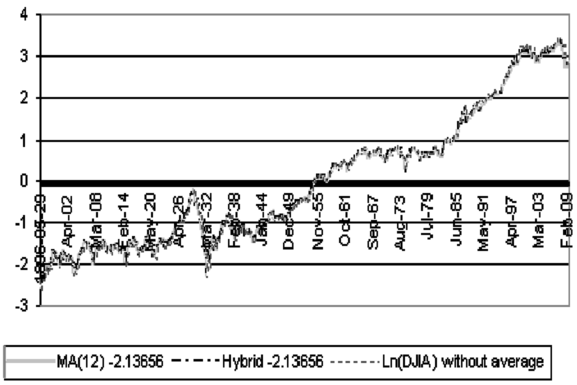

Figure 9. Models for the transformed DJIA by logarithm, then mean extraction.

Figure 9 reflects the models quality. Although it is not obvious from the plot (given the picture size and the fact that it plots over 1300 values), the hybrid model induced an improvement in prediction performance. Another advantage offered is that it models the residual and helps to predict it, fact that can be crucial for larger horizons predictions.

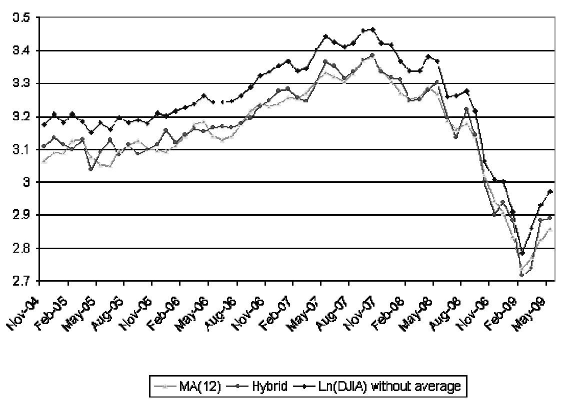

Figure 10. Predicted values for DJIA.

The MSE over the period Nov. 2004- Jul. 2009 (55 months) was improved by 18% by the hybrid by comparison to (17). Considered only over the year 2008 (12 months), the improvement was of only 9%. Contrary to our expectations, the models obtained for the long DJIA are fit. Figure 10 depicts the predictions over the last 55 months in the series.

5. Conclusions

Despite its difficulty, time series forecasting is a problem that attracts much interest from researchers in mathematics, computer science or economy. Although both ARMA and GEP gave good results in problems of model identification, neither of them is an universal modeling technique. While ARMA models offer simple, interpretable formulae [5] that characterize the linear behaviour in data, they rely on certain statistical properties that have to be met in order to obtain a good model.

In addition, the success of an ARMA model depends highly on the level of expertise of the human researcher. On the other hand, GEP models offer the flexibility of modeling series without any constraints imposed on the shape or the size of the model [8]. The expressions derived are, most of the times, complex and need to be processed before being interpretable.

Numerous studies in the literature reported the AR models obtained for stock price time series resemble the random walk model of predicting the next value based on the nearest previous value [5]. The results presented in this paper make no exception. By comparison, our results are consistent with the results presented by Zhang, Pai and Lin in [15, 22]. For every series studied, the models followed the same path. Every time, the shape of the ARMA model is a good fit to the original, with a delay of one time interval. Yet, these models are far from being the most useful in a forecasting scenario.

GEP helps with forecasting since it models the differenced series obtained by removing the autoregressive part from the original data. GEP derives models that detect the nonlinearities in the series and offer analytical even though quite complex formulae that fit well the residuals and help the hybrid achieve predictions better than pure ARMA models.

The hybrid models reflect both linear and nonlinear patterns in the time series. The empirical results with real stock data confirm the hybrid approach to be a fair competitor for the classical methods for modeling time series.

Future work will focus on the dynamic aspect of the data that comes from the financial domain. Thus, procedures to identify breaks in the series will be used and the subsequent segments will be modelled. An important direction of our study is to devise a method that adaptively combines change point detection methods and modeling techniques in order to provide better forecasts on the short run, and moreover, to predict important structural changes in the data generating process of a time series.

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