Explicitly Symplectic Algorithm for Long-time Simulation of Ultra-flexible Cloth

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Abstract: *In this paper, a symplectic structure-preserved algorithm is presented to solve Hamiltonian dynamic model of ultra-flexible cloth simulation with high computation stability. Our method can preserve the conserved quantity of a Hamiltonian, which enables a long-time stable simulation of ultra-flexible cloth. Firstly, the dynamic equation of ultra-flexible cloth simulation is transferred into Hamiltonian system which is slightly perturbed from the original one, but with generalized structure preservability. Secondly, semi-implicit symplecticRunge-Kutta and Euler algorithms are constructed, and able to be converted into explicit algorithms for the separable dynamic models. Thirdly, in order to show the advantages, the presented algorithms are utilized to solve a conservative system which is the primary ultra-flexible cloth model unit. The results show that the presented algorithms can preserve the system energy constant and can give the exact results even at large time-step, however the ordinary non-symplectic explicit methodsexhabit large error with the increasing of time-step. Finally, the presented algorithms are adopted to simulate a large-areaultra-flexible cloth to validate the computation capability and stability. The method employs the symplectial features and analytically integrates the force for better stability and accuracy while keeping the integration scheme is still explicit. Experiment results show that our symplectic schemes are more powerful for integrating Hamiltonian systems than non-symplectic methods. Our method is a common scheme for physically based system to simultaneously maintain real-time and long-time simulation.It has been implemented in the scene building platform-World Max Studio.*

Keywords: *Flexible cloth simulation, numerical integration, symplectic method, scene building system.*

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1. Introduction

The wide range of ultra-flexible cloth in daily-life has motivated much computer graphics research on cloth modelling and rendering [1]. In computer animation especially in digital entertainment, 3D games, fashion design and garment simulation for prototyping applications [12], cloth simulation plays an essential role in increasing the reality of animation. The real-time and long-time animation of ultra-flexible cloth in interactive application has attracted research efforts for the past two decades [21]. The current integration algorithms are unstable in the field of long-time cloth simulation, where the simulation error becomes uncontrollable due to the 'numerical damping'. On the other hand, the real-time simulation needs high computation efficiency which can be realized by larger time steps or less iterations. The implicit symplecticintegration methods were investigated in order to perform high precision simulation and stability, but with low computation efficiency. An extensive review is detailed by Chen *et al*. [3]. It is significant to construct an algorithm for real-time and long-time simulation with low computational cost.

In this paper, we propose an explicitsymplectic Runge-Kuttamethod for long-term stable simulation of ultra-flexiblecloth with low computational cost. The presented algorithm can preserve geometric properties of physically featured model of ultra-flexible cloth for long-time simulation. Firstly, the mass-spring model is employed to simulate the dynamic behaviour of ultra-flexible cloth.

Secondly, the dynamic model is introduced into Hamiltonian systems to maintain the geometric structure. Thirdly, the proposed symplectic integration method is constructed for Hamiltonian systems to keep the long-time computation stability. These algorithms avoid introducing error into the final results no matter how precise the established model is. This may change the dynamic motion of the ultra-flexible cloth. The problem is even serious with stiffness of the mechanical system and the size of the time step [22].

Finally, the improved symplectic numerical integration algorithm and examples in ultra-flexible cloth implemented in the scene building platform-World Max Studio (developed by Institute of Virtual Reality and Visualization Technology College of Information Science and Technology Beijing Normal University). The results given to show our method is more stable and efficient. The aim of this paper is to realize the long-term behaviour in addition to simply obtaining good local accuracy.

2. Related Work

Simulations of ultra-flexibleclothта bend without stretching find applications in computer animation [2, 5], and fashion design. Summaries covering much of the work done in graphics for modellingultra-flexiblecloth can be found in the recent survey articles of Magnenat and Volino [12] and Nealen *et al*. [14]. Obtaining realistic material behaviours typically requires constitutive parameters that result in numerically stiff systems.

Compared to physically consistent models derived from continuum mechanics using the finite element method, the mass-spring systems are simpler, but considered to be easier and precise for implementation. For a more detailed discussion we recommend the survey article [14]. An alternative to classical force-based physics is Position Based Dynamics (PBD) [13], which is unconditionally stable, controllable and faster than traditional simulation methods. The spring projection concept of PBD is closely related to strain limiting [3, 20, 23]. Spring projection concept of PBD is found in the Nucleus system. However, position-based methods are generally not so accurate as traditional ones but provide visual plausibility.

Nomatter whether a mass-spring system or continuum mechanics based method is adopted, a numerical time integration technique is necessary to calculate the dynamic behavior. Explicit time integration methods are fast and flexible but they have stability problems and are prone to failure to long-time simulation. Implicit integration methods can speed up the convergence but require solving large systems of equations, with high computational cost [11]. The most straightforward integration methods are explicit, such as explicit Euler and Runge-Kutta methods to solve the equation of motion [3]. Unfortunately, these approaches cannot keep long-time stability. With the work of Baraff and Witkin, implicit time stepping underwent a stable simulation. However, the computational cost is much higher. A drawback of implicit method is that integration error manifests as excessive numerical damping. A related time-stepping strategy involves the combination of implicit and explicit methods (IMEX) [1, 17]. Geometric integration methods-numerical methods that preserve geometric properties of the flow of a differential equation outperform other schemes (e.g., fourth-order explicit Runge–Kutta method) in predicting the long-term qualitative behaviors of the original dynamic system. Symplectic algorithms based on splitting, such as the leap-frog method for separable Hamiltonian systems, are fast and simple and give very good results for long simulations. Symplectic integrators [7, 18, 19], are known for their energy conservation properties. In general, with an approximation based on a finite number of iterations, the resulting integration scheme is no longer symplectic. Error analysis on the structural conservation, like the analysis on the numerical accuracy, provides insight into a numerical method and helps in making judicious choices of integration schemes. The question is how to strike the right balance between the computational cost and the structural preservation. We extend the applications of these algorithms from accelerator physics, molecular dynamics and celestial mechanics toultra-flexiblecloth. Numerical comparisons were also conducted between several symplectic methods [9].

We can recast the traditional way of thinking about an object accelerating in response to applied forces into a geometric viewpoint of particular interest in computer animation especially in flexible structures simulation. Physically based simulation of ultra-flexiblecloth is computationally expensive, which has been addressed in [17]. The symplectic integrate method is designed and implemented to solve the problem of instability of flexible structures simulation.

Our method enables fast approximate simulation because of mass-spring model is used while symplectic integrated method is adopted for symplectic structure that is conserved to assure the stability of flexible structures simulation.

3. Dynamic Modelling for the Cloth

3.1. The Discrete Dynamic Model of Cloth

Cloth can be represented as a large-area ultra-flexible structures whose mechanical behavior is composed of comparable small in-plane stretching and compression and significant out-of-plane twisting and bending. The mechanical properties of ultra-flexible cloth (stretching, shearing, and bending) are modelled by using spring system. Stretching is the displacement of the ultra-flexible cloth along the warps and the wefts directions. Highly accurate material modelling is not always necessary for physics-based animation of ultra-flexible cloth, so mass-spring-damping systems are widely used for one and two-dimensional structures, such as hairand cloth, and to a lesser extent for elastic solids [17]. As shown in Figure 1, the left is the simulation result while the right is the discrete structure of the right ultra-flexible cloth.

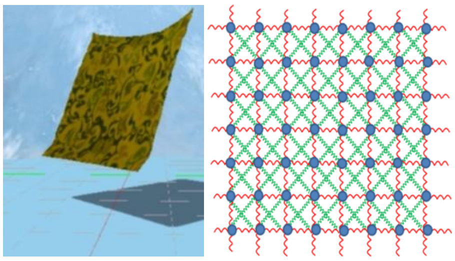


Figure 1. Physical model of cloth and the discontinuous structure of mechanics model. The red and blue spring represent stretch and shear elastic forces separately.

We will consider that a mass point i is linked to the others with springs of rest length l_{ij}^0 and stiffness k_{ij}.

This stiffness value is set to zero if the actual model does not contain a spring between masses i and j. The spring in the clothing model is separated along the warp direction and the weft direction. According to the deformation, the springs of clothing model can be divided into three kinds that are stretch spring, bend spring and shear spring.

We assume a mechanical system with n points, evolving through a discrete set of time points t_1, t_2,…. with equal time interval h. Suppose the system configuration at time t_i as $q_n \epsilon R^{3n}$.The system evolves in time according to physical laws of motion, where forces are represented by a non-linear function of f: $R^{3n} \to R^{3n}$, so that $f(q_n)$ is the vector of forces acting on all particles at time t_i. Only the position dependent forces are considered and the damping forces are ignored. The forces supposed to be conservative, i.e., $f = -\nabla E$, where E:$R^{3n\to}$ R is a potential function, encompassing both internal and external forces. The main task is to calculate system states q_1,q_2,....,when the mass-matrix $M \epsilon R^{3n\times 3n}$ is given.

3.2. Hamiltonian Canonical Dynamic Model

Integration of an integrable Hamiltonian system, where the solution of Newton's equations is reducible to the solution of a set of simultaneous equations, followed by integration over one single variable, will cause a nonintegrable perturbation to the system because the trajectories lie on invariant tori. Here, we describe a scheme for time integration of mass-spring systems, making use of a solver based on semi-implicit Huang *et al*. [6]. The critical step is to transfer the dynamic model into the Hamiltonian model. Here, two degrees of freedom system is adopted to show the process of establishment of Hamiltonian model. Supposed the Lagrangianis $\Psi(q,\dot{q},t)$=$T-V$, where T is kinetic energy and V is potential energy of the mass-spring-damping system.

Define a general momentum as Equation (1) shows:

$$p(t) = \frac{\partial \Psi(q,\dot{q},t)}{\partial \dot{q}} = M(t)\dot{q} \tag{1}$$

Where, $p(t) \in R^n$, n is the number of degree of freedom.

According to Legendre transformation, the Hamiltonian $H(p, q, t)$ can be gotten from the Lagrangian directly, as follows:

$$\begin{aligned} H(p,q,t) &= p^T(t)\dot{q}(t) - \Psi(q,\dot{q}(p,q),t) \\ &= T(p,q,t) + V(q,t) \\ &= \frac{1}{2} p^T(t) M^{-1}(q,t) p(t) + V(q,t) \end{aligned} \tag{2}$$

Then the Hamilton canonical equations of coupled system are expressed as:

$$\dot{q} = \frac{\partial H}{\partial p} = H_p \tag{3}$$

$$\dot{p} = -\frac{\partial H}{\partial q} = H_q \tag{4}$$

$$\frac{\partial L}{\partial t} = -\frac{\partial H}{\partial t} \tag{5}$$

Where, the generalized displacement vector q=[$q_1(t)$, $q_2(t)$, …, $q_n(t)$]^T, the generalized momentum vector p=[$p_1(t)$,$p_2(t)$,…, $p_n(t)$]^T, and

$$H_p = \left[\frac{\partial H}{\partial q_1}, \frac{\partial H}{\partial q_2}, ..., \frac{\partial H}{\partial q_n}\right]^T, \; H_q = \left[\frac{\partial H}{\partial p_1}, \frac{\partial H}{\partial p_2}, ..., \frac{\partial H}{\partial p_n}\right]^T \tag{6}$$

The mass matrix is a function of the generalized coordination when there is large deformation in simulation, namely the mass distribution of the cloth continually changes with developing of the simulation.

Considering the mass matrix is the function of the general coordination, we can define

$$M_q = \begin{bmatrix} \frac{\partial M}{q_1} & \frac{\partial M}{q_2} & \cdots & \frac{\partial M}{q_n} \end{bmatrix} \tag{7}$$

Then, we can get

$$\begin{bmatrix} \frac{\partial M}{q_1} H_p & \frac{\partial M}{q_2} H_p & \cdots & \frac{\partial M}{q_n} H_p \end{bmatrix} = M_q \left(I \otimes H_p \right) \overset{def}{=} F \tag{8}$$

Where $I \in R^{n\times n}$ identity matrix, $\otimes$ is Kronecker operator.

Then the Hamilton canonical equation is transformed into another expression

$$\dot{q} = H_p = M^{-1} p \tag{9}$$

$$\dot{p} = -H_q = \frac{1}{2} F^T H_p - \frac{\partial V}{\partial q} \tag{10}$$

The Equations (9) and (10) have defined a Hamilton vecto

$$G = \left\{ (q,p) \mid \dot{q} = H_p = M^{-1}p, \dot{p} = -H_q = \frac{1}{2} F^T H_p - \frac{\partial V}{\partial q} \right\}.$$

The Lagrange dynamics system is directly transferred into Hamilton dynamics system. Without loss of generality, the Lagrange dynamics system is given as follows:

$$M\ddot{q} + G\dot{q} + Kq = f \tag{11}$$

Where, $M \in R^{n\times n}$, $G \in R^{n\times n}$, $q \in R^n$, $f \in R^n$.

Transform Equation (11) from the Lagrange system into the Hamilton system, we can get

$$\dot{z} = Hz + u \tag{12}$$

where, $z = \begin{bmatrix} q \\ p \end{bmatrix}$, $p = M\dot{q} + Gq/2$, $H = \begin{bmatrix} A & D \\ B & C \end{bmatrix}$, $A = -\frac{M^{-1}G}{2}$, $B = \frac{GM^{-1}G}{4} - K$, $C = -\frac{GM^{-1}}{2}$, $D = M^{-1}$, $u = \begin{bmatrix} 0 \\ f \end{bmatrix}$, $H \in R^{2n\times 2n}$, $z \in R^{2n\times 1}$, $u \in R^{2n\times 1}$ [8].

There exists symplecticeigen value problem when H is Hamiltonian matrix. Supposed that the eigenvalue of matrix H is given, namely

$$H \begin{Bmatrix} q \\ p \end{Bmatrix} = \mu \begin{Bmatrix} q \\ p \end{Bmatrix} \tag{13}$$

According to the features of Hamiltonian matrix, further one can get

$$H^T \begin{Bmatrix} p \\ -q \end{Bmatrix} = -\mu \begin{Bmatrix} p \\ -q \end{Bmatrix} \tag{14}$$

It shows that the eigenvalues appears in pair that one half is located at right side of the imaginary axis, and the rest at the left. There are n pair eigenvalues (μ_i, μ_{n+i}) and eigenvectors (φ_i, φ_{n+i}). The eigenvectors show the following relationship

$$\begin{cases} \varphi_i^T J \varphi_j = 0, \ if \ j \neq \mathrm{mod}_n(n+i) \\ \varphi_i^T J \varphi_j = 1, \ if \ j = \mathrm{mod}_n(n+i) \end{cases} \tag{15}$$

4. Semi-implicit SymplecticAlgorithm

It is well known that Hamilton mechanics cannot be investigated without symplectic difference methods.

The traditional Runge-Kutta algorithm does not fit for Hamilton mechanics, since nonsymplectic algorithm cannot preserve the symplectic form. When the Hamiltonian has a separable structure, i.e., $H(q, p)=T(p)+V(q)$, the semi-implicit Runge-Kutta type algorithms can be transferred into an explicit one which preserves the symplectic structure [4, 10]. In the following, we show that symplectic geometry is the mathematical framework of Hamiltonian system. So, Hamiltonian algorithm should be originated from the framework of symplecticgeometry [8].

The Equation (6) can be expressed:

$$\dot{z} = J_{2n} H_z = g(z) \tag{16}$$

There are two methods to establish difference algorithm of the Hamilton system: generating functions or directly constructing SymplecticRunge-Kutta method.

According to the physically featured and constrained ultra-flexible cloth, we construct one step s-order semi-implicit symplecticRunge-Kutta method as follows:

$$z_{k+1} = z_k + h\sum_{i=1}^{s} b_i g(Y_i) \tag{17}$$

$$Y_i = z_k + h\sum_{j=1}^{s} a_{ij} g(Y_j), (i \le i \le s) \tag{18}$$

Where, $h = t_{k+1} - t_k (k \ge 0)$ .Every difference scheme, explicit or implicit can be regarded as a mapping from time t^i to time t^{i+1}. If the mapping is symplectic, we call the difference scheme symplectic scheme. Whether the general Runge-Kutta algorithm orsymplecticRunge-Kutta algorithm is determined by $D=(a_{ij})$ and $b=[b_i]$.

Or Butcher style can be utilized to express the co-efficient of the symplectic Runge-Kutta method as follows.

$$\begin{array}{c|ccc} c_1 & a_{11} & \cdots & a_{1s} \\ \cdots & \cdots & \cdots & \cdots \\ c_s & a_{s1} & \cdots & a_{ss} \\ \hline & b_1 & \cdots & b_s \end{array} \tag{19}$$

Where, $c_i = \sum_{j=1}^{s} a_{ij}, i = 1, 2, ..., s$. The coefficient of symplectic Runge-Kutta method can be determined by the above Butcher table.

Supposed $B=diag[b_i]$ and $W = BD + D'B - bb'$, the general Runge-Kutta algorithm is symplectic Runge-Kutta algorithm if $W=0$ [23].

The format of Runge-Kutta method is determined by coefficient a_{ij}.

1) if $j \ge i$, $\forall a_{ij} = 0$, $Y_1 = z_k$, and Y_i can be gotten from $Y_1, ... Y_{i-1}$ explicitly. It is an explicit Runge-Kutta method, namely conventional Runge-Kutta method.
2) if $j > i$, $\forall a_{ij} = 0$ and $\exists a_{ii} \neq 0$, and one can get $Y_i = z_k + h\sum_{j=1}^{i-1} a_{ij} g(Y_j) + a_{ii} g(Y_i)$ which is so-called semi-implicit Runge-Kutta method.
3) When the coefficient a_{ij} does not meet the above two condition, it is implicit Runge-Kutta.

According to the definition Equation (16), the following methodsare symplectic methods.

1) Euler midpoint formula (1-order Gauss-Legendre method)

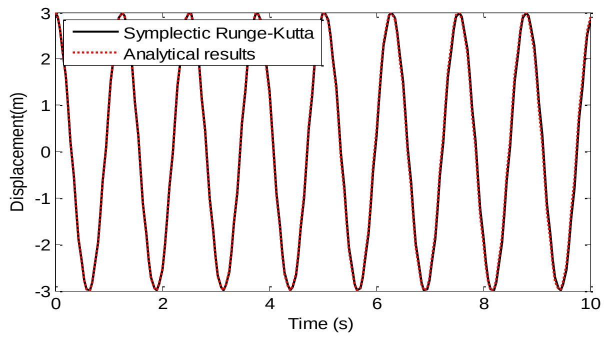
$$\begin{array}{c|c} 1/2 & 1/2 \\ \hline 0 & 1 \end{array} \tag{20}$$

2) 4-order 2-step implicit Runge-Kutta method (2-order Gauss-Legendre method)

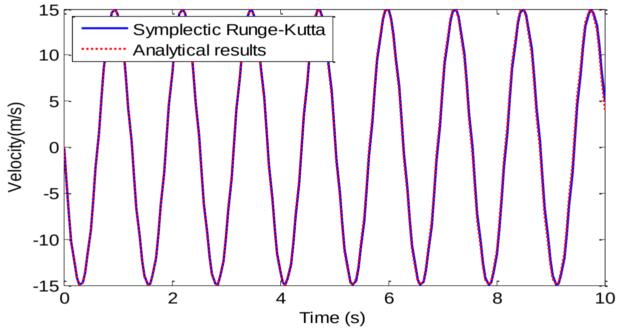
$$\begin{array}{c|cc} \frac{1}{4} - \frac{1}{6}\sqrt{3} & \frac{1}{4} & \frac{1}{4} - \frac{1}{6}\sqrt{3} \\ \frac{1}{2} + \frac{1}{6}\sqrt{3} & \frac{1}{4} + \frac{1}{6}\sqrt{3} & \frac{1}{4} \\ \hline & \frac{1}{2} & \frac{1}{2} \end{array} \tag{21}$$

5. Results and Discussion

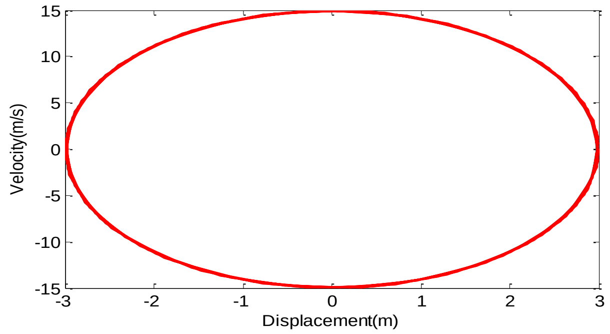
The presented symplectic method is adopted to calculate the dynamic system with the conservation of mechanical energy [16]. In order to show the advantages of symplectic algorithms [3, 10, 15], the computation step of 0.05s is adopted for the two forms. These results are shown in Figure 2. It can be noted from Figure 2-a and Figure 2-b that the amplitudes of the displacement and velocity keep constant. We also adopt the computation step 0.1s and 0.01 to calculate the dynamic equation. In the case of 0.01s and the former three methods are able to get the right results in short term simulation, but the fourth one still gives the results with large error. In Figures 2 and 3, when the computation step 0.1s is adopted, after 200 steps, only the symplectic algorithms can carry out the right simulation, but the results from the other nonsymplectic methods are unable to be convergent to the analytical results.



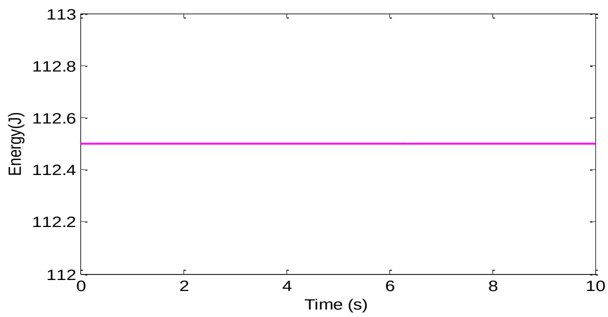
a) time history curve of displacement.



b) time history curve of velocity.

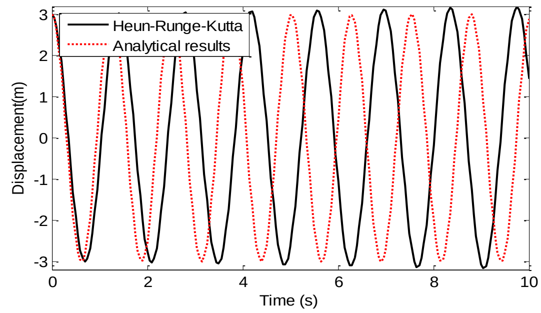


c) phase diagram.

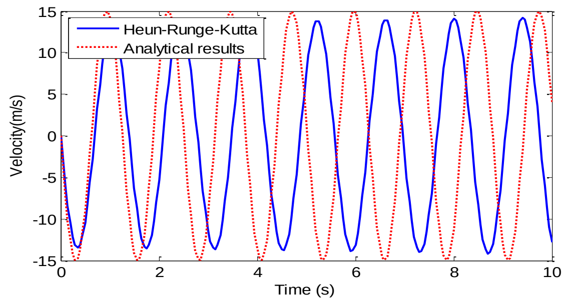


d) time history curve of energy.

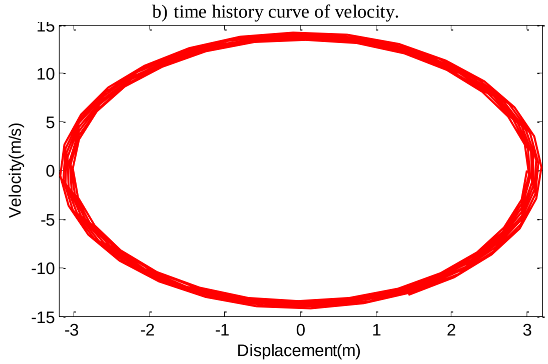
Figure 2. 2-order 2-step semi-implicit symplectic Runge-Kutta scheme after 200 steps.



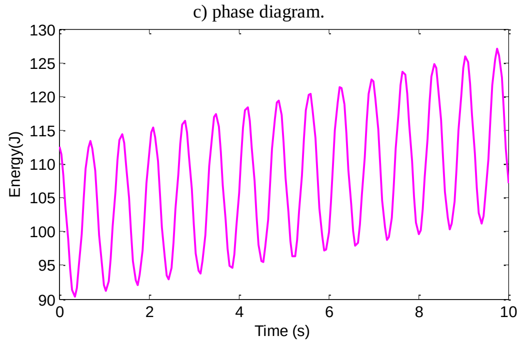
a) time history curve of displacement.



b) time history curve of velocity.



c) phase diagram.

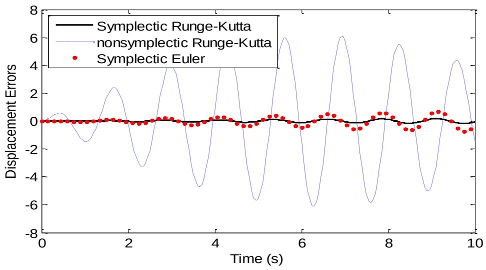


d) time history curve of energy.

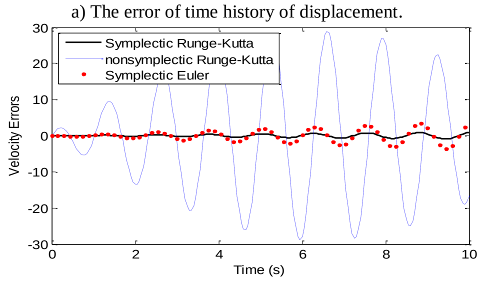
Figure 3. The computation results by Heun-Runge-Kutta method after 200 steps.

In addition to the computation stability of long-term simulation, the precision is also very critical for local results of cloth simulation practically. The precision can be analysed by comparing the numerical results with the analytical result. For dynamic system of Hamiltonian $H=\frac{1}{2}(p^2+k^2q^2)$, the analytical result is $q(t)=\sqrt{q_0^2+\frac{p_0^2}{k^2}}\cos\left(kt-arctg\left(\frac{p_0}{kq_0}\right)\right)$, based on which the displacement, velocity and energy can be derivated directly. Figure 4 shows the errors of displacement, velocity and energy between the numerical results and the analytical results, namely the numerical results minus the analytical results. Here, only symplectic Runge-Kutta scheme, Heun-Runge-Kutta scheme, and

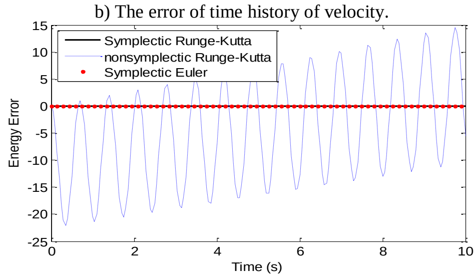
Euler midpoint scheme are compared with the analytical result, because the energy error resulted from the general Euler scheme is too large to be plotted with the other three schemes. It can be found that the errors are very different. The errors from symplectic schemes are far smaller than that of Heun-Runge-Kutta scheme, so that errors from symplectic schemes can be ignored. One can know that the number of order of symplectic scheme plays a critical role in the precision of computation. The higher the order is, the higher the precision is. Although the energy error so small that it can be ignored in practice, there is a further point about symplectic maps that affects all numerical methods using floating-point arithmetic, and that is round-off error. Round-off error is a particular problem for Hamiltonian systems, because it introduces non-Hamiltonian perturbations despite the use of symplectic integrators. The fact that symplectic methods do produce behaviour that looks Hamiltonian shows that the non-Hamiltonian perturbations are much smaller than those introduced by non symplectic methods.



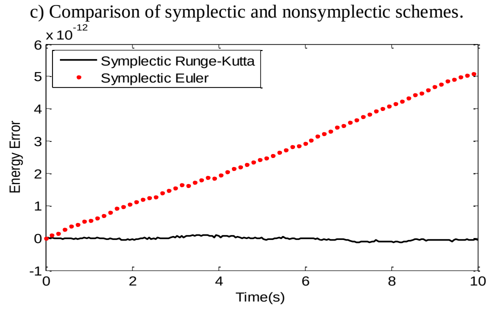
a) The error of time history of displacement.



b) The error of time history of velocity.



c) Comparison of symplectic and nonsymplectic schemes.



d) Comparison of symplectic schemes with different orders.

Figure 4. Time history of energy errors.

The presented symplectic method has been implementedin the scene building platform - World Max Studio (developed by Institute of Virtual Reality and Visualization Technology College of Information Science and Technology Beijing Normal University).

The prototype cloth simulation system uses C++ and DirectX. The simulation results have been compared with the results from the conventional integrated methods and the results show that the presented algorithms can preserve the system energy constant and can give right results even at large time-step. We implemented the method on a system with Intel 2.20 GHz i7 CPU, 4 GB RAM, and Geforce 610M runningonWindows 7 OS.To get an unbiased evaluation of the results, we set the simulation scale the same. The test used for the method is a rectangle cloth surface, initially horizontal, attached along one of its edges. The material density is 13.78g/m^2 and has a weft and warp elasticity modeled by springs roughly corresponding to the tensile properties of a soft shirt fabric. We design experiments to test how the proposed symplectic Runge-Kutta has more advantages than the classical non-symplectic Euler and Runge-Kutta.

We designed the following experiment to show the energy conservation of our method. The result shown in Figure 5 of upper is implemented with the Euler method, and the bottom is the result produced by our method. For both simulations, the stiffness is set to be 50.75.In both cases, we use constant time-step of 16.7 milliseconds (60 iterations per second according to the most popular physics engine such as Bullet).We compare the simulation of the un damped cloth using Euler with the simulation using Symplectic Euler, as shown in the Figure 5, the accuracy of Euler method is not satisfactory, whereas our method shows stable and reason able dynamic behaviors. As for implicit integrate method, because the simulation time is too long to compare with these explicit or semi-implicit ones, we omit the results of implicit method.

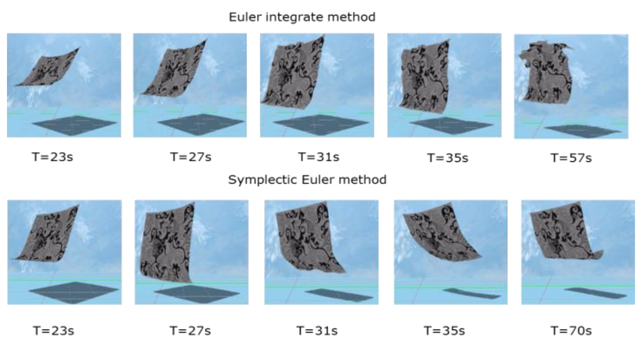


Figure 5. The simulation result with no damping with explicit Euler method and Symplectic Euler method.The presented algorithm is implemented in the scene building platform - World Max Studio (developed by Institute of Virtual Reality and Visualization Technology College of Information Science and Technology Beijing Normal University).

6. Conclusions

In this paper, we propose a symplectic Euler method to solve Hamiltonian dynamic model of ultra-flexible cloth with constraint. The proposed method is based on the Hamiltonian dynamic model and simulates the mass-spring model with improved stability and accuracy while keeping the integration scheme still explicit. The proposed semi-implicit symplectic Runge-Kutta and Euler algorithms are derived and can be transformed to explicit algorithms since the dynamic model is separable. The experiments showed it is an efficient and stable method for animating and rendering mass-spring-based complex cloth models for interactive applications, enabling the stable simulation of cloth in long real-time. The proposed method can be applied to real-time application for efficient animation and rendering of realistic virtual cloth.

In the following research, the method will be used in other cloth model other than mass-spring model.

Because the integration scheme is explicit, the method can be parallelized, and the performance improvement by exploiting the parallelism in GPU can be obtained.

References

[1] Bridson R., Marino S., and Fedkiw R., "Simulation of Clothing with Folds and Wrinkles," *in Proceedings of the ACM SIGGRAPH/Eurographics Symposium on Computer Animation*, San Diego, pp. 28-36, 2003.

[2] Chen S., "Human Visual Perception-based Image Quality Analyzer for Assessment of Contrast," *The International Arab Journal of Information Technology*, vol. 13, no. 2, pp. 238-245, 2016.

[3] Chen Y., He B., and Yu X., "Research on Garment Wrinkle Synthetic Method Based on Mass-Spring Model," *Journal of Electroanalytical Chemistry*, vol. 621, no. 2, pp. 121-123, 2015.

[4] Choi K. and Ko H., "Stable but Responsive Cloth," *in Proceedings of the 29th Annual Conference on Computer Graphics and Interactive Techniques*, San Antonio, pp. 604-611, 2005.

[5] Hauth M., Etzmuss O., and Strasser W., "Analysis of Numerical Methods for the Simulation of Deformable Models," *The Visual Computer*, vol. 19, no. 7-8, pp. 581-600, 2003.

[6] Huang Y., Yin Z., and Xiong Y., "Modeling and Computation for the High-Speed Rotating Flexible Structure," *Journal of Vibration and Acoustics*, vol. 130, no. 4, pp. 2727-2747, 2008.

[7] Huang Y., Deng Z., and Yao L., "An improved Symplectic Precise Integration Method for Analysis of the Rotating Rigid-flexible Coupled System," *Journal of Sound and Vibration*, vol. 299, no. 1-2, pp. 229-246, 2007.

[8] Huang Y., Deng Z., Xiong Y., "High-order Model and Slide Mode Control for Rotating Flexible Smart Structure," *Mechanism and Machine Theory*, vol. 43, no. 8, pp. 1038-1054, 2008.

[9] Huang Y., Deng Z., and Li W., "Sliding Mode Control based on Neural Network for the Vibration Reduction of Flexible Structures," *Structural Engineering and Mechanics*, vol. 26, no. 4, pp. 377-392, 2007.

[10] Huang Y., Deng Z., and Yao L., "Dynamic Analysis of a Rotating Rigid-flexible Coupled Smart Structure with Large Deformations," *Applied Mathematics and Mechanics*, vol. 28, no. 10, pp. 1349-1360, 2007.

[11] Liu L., Wang R., Su Z., Luo X., and Gao C., "Mesh-based Anisotropic Cloth Deformation for Virtual Fitting," *Multimedia Tools and Applications*, vol. 71, no. 2, pp. 411-433, 2014.

[12] Magnenat N. and Volino P., "From Early Draping to Haute Couture Models: 20 Years of Research," *The Visual Computer*, vol. 21, no. 8-10, pp. 506-519, 2005.

[13] Müller M., Heidelberger B., Hennix M., and Ratcliff J., "Position based Dynamics," *Journal of Visual Communication and Image Representation*, vol. 18, no. 2, pp. 109-118, 2007.

[14] Nealen A., Müller M., Keiser R., Bxerman E., and Carlson M., *Physically based Deformable Models in Computer Graphics*, Wiley Online Library, 2006.

[15] Qin Y., Deng Z., and Hu W., "Structure-Preserving Algorithm for Steady-State Solution to the Infinite Dimensional Hamilton System," *Applied Mathematics and Mechanics*, vol. 35, no. 1, pp. 22-28, 2014.

[16] Selle A., Lentine M., and Fedkiw R., "A mass Spring Model for Hair Simulation," *ACM Transactions on Graphics*, vol. 27, no. 3, pp. 15-19, 2008.

[17] Stam J., "Towards a Unified Dynamical Solver for Computer Graphics," *Computer Graphics Forum*, vol. 25, no. 3, pp. 232-242, 2006.

[18] Stern A. and Desbrun M., "Discrete geometric Mechanics for Variational Time Integrators," *in Proceedings of ACM SIGGRAPH Courses*, Boston, pp. 75-80, 2006.

[19] Stern A. and Grinspun E.,"Implicit-explicit Variational Integration of Highly Oscillatory Problems," *Multiscale Modeling and Simulation*, vol. 7, no. 4, pp. 1779-1794, 2009.

[20] Thomaszewski B., Pabsts S., and Straber W., "Continuum−based Strain Limiting," *Computer Graphics Forum*, vol. 28, no. 2, pp. 569-576, 2009.

[21] Volino P. and Magnenat-Thalmann N., "Accurate Garment Prototyping and Simulation," *Computer-Aided Design and Applications,* vol. 2, no. 5, pp. 645-654, 2005.

[22] Volino P. and Magnenat-Thalmann N., "Comparing Efficiency of Integration Methods for Cloth Simulation," *in Proceedings of Computer Graphics International*, Hong Kong, pp. 265-272, 2001.

[23] Wang H., O'Brien J., and Ramamoorthi R., "Multi-Resolution Isotropic Strain Limiting," *ACM Transactions on Graphics*, vol. 29, no. 6, pp. 156, 2010.

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