Image Segmentation and Edge Detection Based on Chan-Vese Algorithm

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Abstract: *The main idea in this paper is to detect regions (objects) and their boundaries, and to isolate and extract individual components from a medical image. This can be done using K-means firstly to detect regions in a given image. Then based on techniques of curve evolution, Chan-Vese for segmentation and level sets approaches to detect the edges around each selected region. Once we classified our images into different intensity regions based on K-means method, to facilitate separating each region with its boundary and its area individually in the next steps. Then we detect regions whose boundaries are not necessarily defined by gradient using Chan-Vese algorithm for segmentation. In the level set formulation, the problem becomes a mean-curvature flow like evolving the active contour, which will stop on the desired boundary of our selected region which results from K-means step. The final image segmentation results are one closed boundary per actual region in the image and a segmented map.*

Keywords: *Chan-Vese approach, K-means, active counters, level set methods, segmentation, edge detection.*

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1. Introduction

The shape of an object can be described either in terms of its boundary or in terms of the region it occupies. The shape representation based on boundary information requires image edge detection and edge following. On the region – based approach, shape representation requires image segmentation in several homogeneous regions.

The basic idea in *active contour models or snakes* is to evolve a curve, subject to constraints from a given image, in order to detect objects in that image. For instance, starting with a curve around the object to be detected, the curve moves toward its interior normal and has to stop on the boundary of the object.

In the classical snakes and active contour models (see [2, 3, 4, 6]), an edge-detector is used, depending on the gradient of the image u_0, to stop the evolving curve on the boundary of the desired object. A general edge-detector can be defined by a positive and decreasing function:

$$g: [0, +\infty] \rightarrow R$$

Depending on the gradient of the image u_0, such that

$$[g(0) = 1 \text{ and } g(z) \rightarrow 0 \text{ as } z \rightarrow \infty], \text{ or as:}$$

$$\lim_{z \rightarrow \infty} g(z) = 0$$

For instance:

$$g(|\nabla u_0(x, y)|) = \frac{1}{1 + |\nabla G_\sigma(x, y) * u_0(x, y)|^p}, \quad p \geq 1 \quad (1)$$

Where $G_\sigma(x, y) * u_0(x, y)$, a smoother version of u_0, is the convolution of the image u_0 with the Gaussian $G_\sigma(x, y) = \sigma^{-1/2} e^{-|x^2+y^2|/4\sigma}$. The function $g(|\nabla u_0|)$ is positive inhomogeneous regions, and zero at the edges. That is composed of edge points (maxima values of $|\nabla u_0|$ give minima values for $g(.)$) [9].

In problems of *curve evolution,* the *level set* method and in particular the motion by mean curvature of Osher and Sethian [8] have been used extensively, because it allows for cusps, corners, and automatic topological changes. Moreover, the discretization of the problem is made on a fixed rectangular grid. The curve C is represented implicitly via a Lipschitz function ϕ, by $C = \{(x, y) | \phi(x, y) = 0\}$, and the evolution of the curve is given by the zero-level curve at time t of the function $\phi(t, x, y)$. Evolving the curve C in normal direction with speed F amounts to solve the following differential equation [8]:

$$\frac{\partial \phi}{\partial t} = |\nabla \phi| F, \ \phi(0, x, y) = \phi_0(x, y) \quad (2)$$

Where the set $\{(x, y) | \phi_0(x, y) = 0\}$ defines the initial contour. A particular case is the motion by mean curvature, when $F = div(\nabla \phi(x, y) / |\nabla \phi(x, y)|)$ is the curvature of the level-curve of ϕ passing through (x, y) The equation becomes:

$$\begin{cases} \dfrac{\partial \phi}{\partial t} = |\nabla \phi| div\left(\dfrac{\nabla \phi}{|\nabla \phi|}\right), & t \in (0, \infty), \quad x \in R^2 \\ \phi(0, x, y) = \phi_0(x, y), & x \in R^2 \end{cases} \quad (3)$$

1.1. Geometric Active Contour Models

A *geometric active contour model* based on the mean curvature motion is given by the following evolution equation [2]:

$$\begin{cases} \dfrac{\partial \phi}{\partial t} = g(|\nabla u_0|)|\nabla \phi| \left(div \left(\dfrac{\nabla \phi}{|\nabla \phi|} \right) + v \right), & in\ (0, \infty)\ x\ R^2 \\ \phi(0, x, y) = \phi_0(x, y), & in \quad R^2 \end{cases} \tag{4}$$

Where:

$g(|\nabla u_0|)$: Edge-function with $p = 2$; (see equation (1)).

$v \geq 0$: Is constant.

ϕ_0: Initial level set function.

Its *zero level curve* moves in the normal direction with speed $g(|\nabla u_0|)$ $(curv(\phi)(x, y) + \mu)$ and therefore stops on the desired boundary, where g vanishes [1]. Since all these *classical snakes and active contour models above* rely on the edge-function g, depending on the image gradient $|\nabla u_0|$, to stop the curve evolution, these models can detect only objects with edges defined by gradient. In practice, the discrete gradients are bounded and then the stopping function is never zero on the edges, and the curve may pass through the boundary, especially for the models in [2, 6]. If the image u_0 is very noisy, then the isotropic smoothing Gaussian has to be strong, which will smooth the edges too. In this paper, we used a different active contour model based on Chan-Vese and K-means algorithms, i.e. a model which is not based on the gradient of the image u_0 for the stopping process.

1.2. Our Procedures

- *First step*: We used K-means algorithm [12] to classify our image into different intensity regions. The basic idea of K-means clustering method is to find a partition (S_j) of the data points that minimizes the sum of squared distance to the center of the cluster. This can be achieved as follows:

  1. Points are assigned at random into K sets S_j.
  2. Each point is assigned to the set whose mean center is the closest. This is repeated until no point changes of set S_j.

  The initial mean intensity value of each region in the image was defined according to the image histogram, where the locations of peaks and valleys of a histogram indicate the clusters of similar-spectral pixels in our image. For more details see our previous work in [10].
- *Second step*: Initial curves are drawn (more than one curve) in the image and then seed point is put inside each curve to be ready initialized by level set in the next step.
- *Third step*: Level set is initialized which can be anywhere in the image.
- *Fourth step*: Chan-Vese approach is used for segmentation.

Our segmentation procedures presented here can be run on Pentium I (300 MHz). But we used – Pentium 4 (2.4 GHz) and VC++ programming under Windows (Xp-2000) to display our results in this paper.

2. Level Curve Example

It is possible graphically to depict a function $f(x, y)$ of two variables using a family of curves called level curves. Let C be any number. Then the graph of the equation $f(x, y) = C$ is a curve in the *xy-plane* called the level curve of a height C. This curve describes all points of height C on the graph of the function $f(x, y)$. As C varies, we have a family of level curves indicating the sets of points on which $f(x, y)$ assumes various values C. in Figure 1-(a, b), we drawn the graph and various level curves for the function $f(x, y) = x^2 + y^2$ [5].

Level curves often have increasing physical interpretations. For example, let (x, y) specifies the coordinates of a point on the earth (x = *latitude*, y = *longitude*) and $f(x, y)$ the current temperature at location (x, y). Then the level curves of the function $f(x, y)$ indicate the locations having equal temperatures, see Figure 1-c. Such curves are called isotherms. In the same way the evolving curve moves toward its interior normal and has to stop on the boundary (edges) of the selected region.

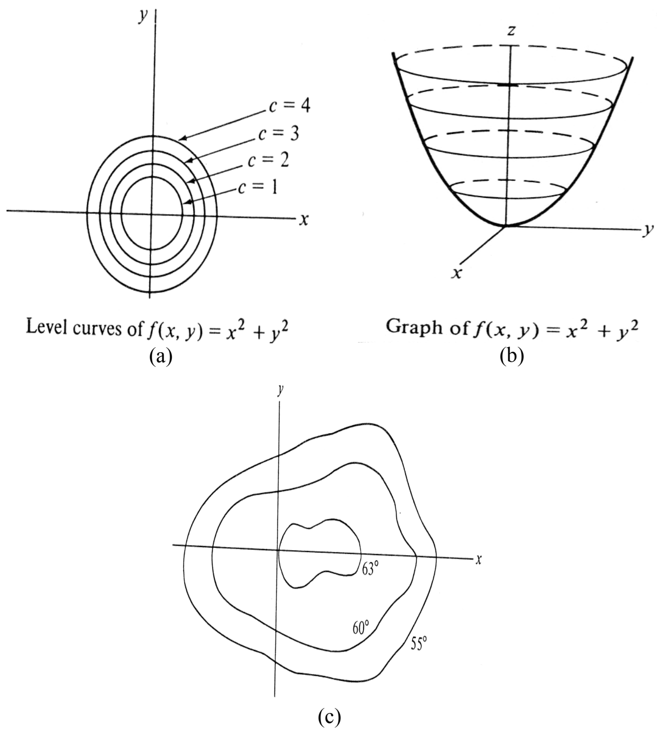


Figure 1. Isotherm and function $f(x, y) = x^2 + y^2$ level curves.

3. Chan-Vese Algorithm

This is closely related to the classical Mumford-Shah algorithm [7], but uses a simple level set framework for its implementation. We present the original Chan-Vese segmentation algorithm [7] which is presented in details in [12]. Therefore for discussing various aspects and details of this algorithm, we refer the reader to References [7, 12].

3.1. Basic Formulation

The minimization problem is:

$$\min_{\phi \in BV(\Omega), c_1, c_2 \in R^+} E(\phi, c_1, c_2; u_0)$$

Where the energy is defined as:

$$E(\phi, c_1, c_2; u_0) = \mu \int_\Omega \delta(\phi) |\nabla \phi| \, dx + \lambda_1 \int_\Omega |u_0 - c_1|^2 H(\phi) \, dx + \lambda_2 \int_\Omega |u_0 - c_2|^2 (1 - H(\phi)) \, dx \tag{5}$$

Intuitively, one can interpret from this energy that each segment is defined as the subregions of the images over which the average of the given image is closest to the image value itself in L_2-norm. The first term in the energy measures the arc length of the segment boundaries. Thus, minimizing this quantity provides stability of the algorithm as well as preventing fractal like boundaries from appearing. If one regularizes the d function and the Heaviside function by two suitable smooth functions δ_ε and H_ε, then formally, the Euler-Lagrange equations can be written as:

$$\partial_\phi E = -\delta_\varepsilon(\phi) \left[\mu \nabla . \frac{\nabla \phi}{|\nabla \phi|} - \nu - \lambda_1 (u_0 - c_1)^2 + \lambda_2 (u_0 - c_2)^2 \right] = 0 \tag{6}$$

With natural boundary condition

$$\frac{\delta_\varepsilon(\phi)}{|\nabla \phi|} \frac{\partial \phi}{\partial \vec{n}} = 0 \quad on \quad \partial\Omega$$

$$c_1(\phi) = \frac{\int_\Omega u_0(x) H_\varepsilon(\phi(x)) \, dx}{\int_\Omega H_\varepsilon(\phi(x)) \, dx} \tag{7}$$

and

$$c_2(\phi) = \frac{\int_\Omega u_0(x)(1 - H_\varepsilon(\phi(x))) \, dx}{\int_\Omega (1 - H_\varepsilon(\phi(x))) \, dx} \tag{8}$$

3.2. Discretization

A common approach to solve the minimization problem is to perform gradient descent on the regularized Euler-Lagrange equation (6); i. e., solving the following time dependent equation to steady state:

$$\frac{\partial \phi}{\partial t} = -\partial_\phi E = \delta_\varepsilon(\phi) \left[\mu \nabla . \frac{\nabla \phi}{|\nabla \phi|} - \nu - \lambda_1 (u_0 - c_1)^2 + \lambda_2 (u_0 - c_2)^2 \right] \tag{9}$$

Here, we remind the readers that $c_1(\phi)$ and $c_2(\phi)$ are defined in equations (7) and (8). In Chan-Vese algorithm, the authors regularized the Heaviside function used in equations (7) and (8) as follows:

$$H_{2,\varepsilon}(z) = \frac{1}{2}\left(1 + \frac{2}{\pi} \arctan(\frac{z}{\varepsilon}) \right)$$

And define the delta function as the derivative of it:

$$\delta_{2,\varepsilon}(z) = H^1_{2,\varepsilon}(z)$$

Equation (9) is then discretized by a semi-implicit scheme; i. e., to advance from $\phi^n_{i,j}$ to $\phi^{n+1}_{i,j}$, the curvature term right hand side of (9) is discretized using the value of $\phi^n_{i_\pm, j_\pm}$, except for the diagonal term $\phi_{i,j}$, which uses the implicitly defined $\phi^{n+1}_{i,j}$. The integrals defining $c_1(\phi)$ and $c_2(\phi)$ are approximated by simple Riemann sum with the regularized Heaviside function defined above. ϕ_t is discretized by the forward Euler method: $(\phi^{n+1}_{i,j} - \phi^n_{i,j})/\Delta t$. Therefore, the final update formula can be conceptually written as:

$$\phi^{n+1}_{i,j} = \frac{1}{1+\alpha_k}(\phi^n_{i,j} + G(\phi^n_{i-1,j}, \phi^n_{i+1,j}, \phi^n_{i,j-1}, \phi^n_{i,j+1})),$$

Where $\alpha_k \geq 0$ comes from the discretization of the curvature term. If the scheme is fully explicit, $\alpha_k = 0$ and G would depend on $\phi^n_{i,j}$. In the paper, the author used $\Delta x = \Delta y = 1$, $\varepsilon = 1$, and $\Delta t = 0.1$. This implies that the delta function is really a regular bump function that puts more weight on the evolution of the zero level set of ϕ. See some results of this algorithm applied to brain segmentation as in [1]. Finally, it is also possible but not advisable in this (unusual) case because new zero level sets are likely to develop spontaneously. On other hand, two distinct approximations and regularizations of the functions H and δ_0 presented in [13] can be used as follows:

$$H_{1,\varepsilon}(z) = \begin{cases} 1 \quad i \quad fz > \varepsilon \\ 0 \quad if \quad z < -\varepsilon \\ \frac{1}{2}\left[1 + \frac{z}{\varepsilon} + \frac{1}{\pi}\sin(\frac{\pi z}{\varepsilon})\right] \quad if \quad |z| \le \varepsilon \end{cases}$$

and

$$H_{2,\varepsilon} = \frac{1}{2}\left[1 + \frac{2}{\pi}\arctan(\frac{z}{\varepsilon})\right]$$

4. Level Sets

The level sets equation given by Osher and Sethian [8] as:

$$\phi_t + F|\nabla\phi| = 0, \qquad given \qquad \phi(x,t=0) \quad (10)$$

This equation describes the time evolution of the level set function *(ϕ)* in such a way that the zero level set of this evolving function is always identified with the propagating see Figure 2-b.

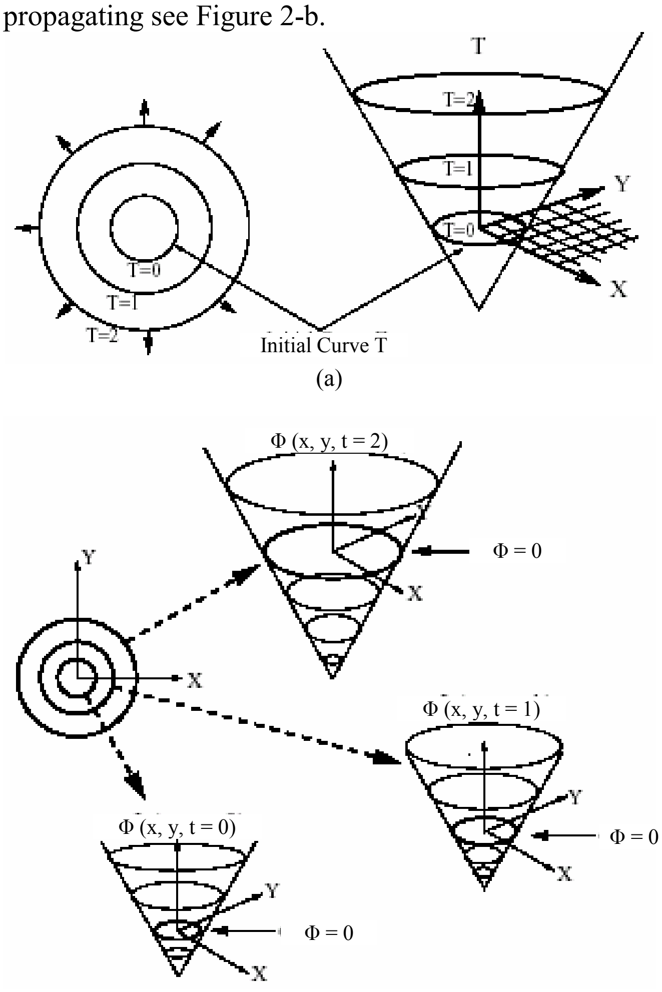


Figure 2. Transformation of front motion: (a) Into boundary value problem, (b) Into initial value problem.

Also in the *level set method* [8], C $\subset$ Ω is represented by the *zero level set* of a *Lipschitz function* ϕ: Ω→R, such that:

$$\begin{cases} C = \partial\omega = \{(x,y) \in \Omega : \phi(x,y) = 0\} \\ inside(C) = \omega = \{(x,y) \in \Omega : \phi(x,y) > 0\} \\ outside(C) = \Omega \setminus \overline{\omega} = \{(x,y) \in \Omega : \phi(x,y) < 0\} \end{cases} \quad (11)$$

Where ω $\subset$ Ω is open, and *C* = *∂ω*. We illustrate in Figure 3 the above assumptions and notations on the level set function *ϕ*, defining the evolving curve *C*. For more details, we refer the readers to Refs.[8, 7, 11].

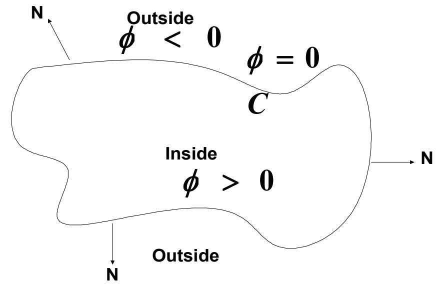


Figure 3. Curve C = {*(x, y): ϕ (x, y)* =} propagating in normal direction.

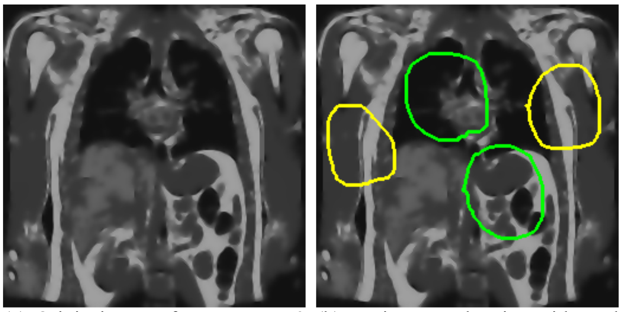
5. Results

In our work, we used an approach derived from initializing a small curve(s) inside the region(s) of interest, see Figure 4-b, and allowing it to grow outwards until it reaches the desired boundary as in Figure 4-d. In this step we can initialize more than one closed curve according to the different intensity regions in our image based on *K-means* model [12], and then we set seed points inside every closed curve represent an intensity value of that region.

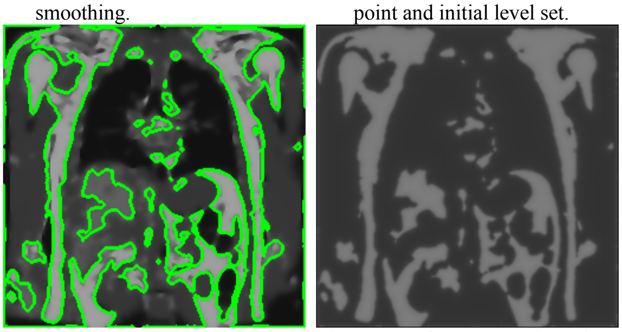
After that we start initialized level set curve to propagate the initial seed point outwards, followed by Chan-Vese method to represent the curve in order to fine tune the result. Finally, we obtain a segmented image with closed boundary per one actual region as in Figure 4-c. We can use one closed curve, but in this case the evolving time and the number of iteration of our algorithm are large. On other hand these procedures can be used with gradient and without gradient images.

Also during our process, we used initial segmented images (different intensity regions) based on *K-means* to superimpose the region boundary and to extract the bounded region (segmented map) in our image as in Figure 4-d as an example. We can use different kind of images to extract different features (*roads, rivers, agricultural areas ...etc*) as in remote sensing images.

So, our results accuracy for edge position depend on the fact that if the results of K-means are accurate then the regions boundaries are in correct position as shown from the figures above. Also, in this method, if we want to choose any region in the image and to define its edge, we can do all that. Then we can calculate some region information such as the area of that region, region map as in Figure 4-d and contour length clearly.



(a) Origin image after K-means & smoothing. (b) Begin curve drawing with seed point and initial level set.



(c) Segmented image by C-Vese approach to get regions with their edges. (d) Map of segmented regions only, of the previous step (c).

Figure 4. Segmented regions with edge detection of abdomen medical image.

The results in Figure 4 are given as an example to test our work in this paper, where we can segment and superimpose accurate edges for other different images with good results of image segmentation and edge detection. Also it is easy to calculate the region area and the boundary length. Our results accuracy for edge position and region segmentation has been compared with the results in [1, 10].We found our image has better region segmentation and edge detection results.

6. Conclusion

Using an active contours based on techniques of curve evolution, *Chan-Vese algorithm for segmentation and level sets is a good and accurate method* to detect object (region) boundaries, to isolate and extract individual components from our digital image. It is possible to detect objects whose boundaries are not necessarily defined by gradient by, where the stopping term does not depend on the gradient of the image, as in the classical active contour.

The initial curve of level set can be anywhere in the image. This help us obtain the final image segmentation is one closed boundary per actual region in the image where the segmentation problem involves finding the closed curve C that lies along the boundary of the object of interest in the image. Then it is easy to calculate the region area and the boundary length, for example. The level set approach allows the evolving front which can extract the boundaries of particularly intricate contours.

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