From Neutrosophic Soft Set to Effective Neutrosophic Soft Set Generalizations and Applications

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Abstract: *The Neutrosophic Soft Set (NSS) is an advanced and highly effective expansion of soft sets, specifically designed to handle parameterized values of alternatives. As an enhanced version of fuzzy soft sets, it provides a novel mathematical framework that offers significant advantages in dealing with uncertain information. This model is created by merging soft sets and neutrosophic sets, providing a robust approach to uncertainty management. Various algorithms have been proposed for making neutrosophic decisions using NSSs. However, these algorithms neglect external effective that influence the Decision-Making (DM) process, focusing solely on parameters. To address this issue, the article introduces the concept of Effective Neutrosophic Soft Sets (ENSSs). Additionally, we extend and generalize the innovative concept of Effective Fuzzy Soft Sets (EFSSs) to accommodate three independent membership criteria, aiming to enhance effectiveness and realism. We also introduce operations on ENSSs, including subset, complement, union, intersection, AND, and OR, which are defined along with illustrative examples. Furthermore, we examine some of its properties. Moreover, we present applications of this concept in DM problems and Medical Diagnosis (MD).*

Keywords: *Soft set, neutrosophic soft set, effective set, effective fuzzy soft set, effective neutrosophic soft set.*

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1. Introduction

Fuzzy set was introduced by Zadeh [31] as a new mathematical tool to deal with uncertain information. Soft Set was introduced by Molodtsov [20]. Practical problems in economics, engineering, medical science, et cetera were solved by soft set.

Maji *et al*. [17] defined some operations like AND, OR and operation of union and intersection. Several researchers, including Maji *et al*. [18], have utilized soft set theory to address Decision-Making (DM) problems. He presented an algorithm for selecting the optimal house using soft set theory.

Numerous researchers have explored the concept of soft set theory and its properties, investigating its applications in various domains. One notable application is found in the work of Roy and Maji [21]. The concept of soft multiset was introduced by Alkhazaleh *et al*. [10]. Majumdar and Samanta [19] introduced the generalized fuzzy soft set, as discussed in their research, which delves into different characteristics and applications related to DM and Medical Diagnosis (MD). A wider understanding of the fuzzy soft set concept entails the consideration of Possibility Fuzzy Soft Sets (PFSSs) [9].

They applied this in DM scenarios and introduced a measure of similarity between two PFSS, which they utilized in MD. Yan [30] introduced and explored various fuzzy data classifications, such as fuzzy basic data types, fuzzy group data types, and fuzzy custom data types.

Smarandache [27] introduced neutrosophy, as outlined in as a novel approach for addressing issues characterized by imprecise, indeterminate, and inconsistent data.

The Neutrosophic Soft Set (NSS) is a hybrid of a soft set and a neutrosophic set, as specified by Maji [16].

In 2013, Broumi and Smarandach [13] introduced the concept of an intuitionistic NSS, detailing its operations and attributes. Neutrosophic sets have been employed to tackle practical DM dilemmas involving uncertain data [14]. Alkhazaleh and Salleh [8] proposed the idea of a soft expert set, which seeks to merge the perspectives of all experts into a cohesive model without requiring operations.

Soft expert set has different extensions and generalization such as; fuzzy soft expert sets [7], neutrosophic soft expert sets [23], generalized neutrosophic soft expert set [29] and Q-Neutrosophic Soft Expert Set (Q-NSES) [15].

The publications referenced in [1, 3, 6, 11, 12] explore the advancements in Neutrosophic Soft Sets (NSSs), particularly focusing on their applications in DM processes and MD.

Smarandache [28] presented a new concept which is hypersoft set which deal with multi-attribute valued function. The fundamentals of hypersoft set discussed by Saeed *et al*. [22] with related basic properties and operations such hypersoft subset, complement, union, intersection and aggregation operators with hypersoft set relation.

Neutrosophic Hyper-Soft Set (NHSS) was introduced by Saqlain *et al*. [24] to overcome the uncertainty problems. He introduces the concept of NHSS with operators, aimed at managing scenarios where attributes need to be subdivided into distinct attribute-valued sets within neutrosophic set contexts. NHSS was generalized into: Interval valued, m-Polar and m-Polar interval valued NHSSs and the operations are discussed with suitable examples by Saqlain and Xin [26]. Zulqarnain *et al*. [32] extended the aggregate operators of the NHSS. They applied these operators to address multi-criteria DM problems, incorporating distance-based similarity measures. Additionally, Saqlain *et al*. [25] introduced a single-valued NHSS, a multi-valued NHSS, and a tangent similarity measure for single-valued NHSSs, along with outlining their properties. He also introduced an algorithm in decision making applying single-valued NHSS dependent on the tangent similarity measure. Another development of NHSS appear in [2].

Alkhazaleh [4] introduced the novel idea of Effective Fuzzy Soft Set (EFSS), delineating its fundamental operations and properties. He elaborated on the impact of external effectiveness on soft sets. Moreover, he illustrated the practical application of this concept in solving problems related to DM with algorithm. Additionally, he provided an algorithm as an enhancement of the one proposed by Roy and Maji [21], and he conducted a comparison between the original and enhanced algorithms. Finally, he provide an application on MD.

Following this, a fresh idea emerged in this field under the title of the Effective Fuzzy Soft Expert Set (EFSES) [5], which combines the strengths of EFSSs and soft expert sets. By expanding upon the concept of EFSSs, we introduce the notion of Effective Neutrosophic Soft Sets (ENSSs) to leverage the advantages of both EFSSs and NSSs. This fusion enhances efficiency and practicality. The concept of EFSSs has been widened to encompass ENSSs, which consider the influence on three distinct membership functions. In this investigation, we introduce the concept of ENSSs. Moreover, we delve into the basic operations and characteristics of this concept using pertinent examples. Lastly, we illustrate an application in DM problems and MD.

2. Preliminary

In this section we recall some definitions required in this paper. Assume that U be a universe, $P(U)$ be the power set of U, E be parameters set and $A \subseteq E$.

- **Definition 1 [20]**

A soft set over the set U is denoted by a pair (F, A) where F represents a mapping from A to the power set of U, i.e., $F: A \rightarrow P(U)$.

- **Definition 2 [21]**

Consider an initial universal set U and a set of parameters E. Let I^U represent the power set of all fuzzy subsets of U. If A is a subset of E, then (F, E) is a fuzzy soft set over U and F is a mapping defined as $F: A \rightarrow I^U$.

- **Definition 3 [27]**

A neutrosophic set A over the universe of discourse X is defined as $A=\{\langle x: T_A(x), I_A(x), F_A(x)\rangle; x \in X\}$, where T_A; I_A; F_A: $X \rightarrow [0,1]$ are functions with the constraint $0 \leq T_A(x)+I_A(x)+F_A(x) \leq 3$. Here, $T_A(x)$ represents the truth-membership function, $I_A(x)$ is the indeterminacy-membership function, and $F_A(x)$ is the falsity-membership function.

- **Definition 4 [16]**

Suppose U is the initial universe set, and E is a set of parameters with $A \subseteq E$. Let $P(U)$ represent the set of all neutrosophic sets of U. The pair (F, A) is referred to as the soft neutrosophic set over U, where F is a mapping defined as $F: A \rightarrow P(U)$.

- **Definition 5 [16]**

Consider two NSSs (F, A) and (G, B) defined over the common universe U. The union of (F, A) and (G, B) is represented as $(F, A) \cup (G, B)$ and is defined as $(F, A) \cup (G, B)=(K, C)$, where $C=A \cup B$. The truth-membership, indeterminacy-membership, and falsity-membership of (K, C) are determined as follows:

$$T_{\mathcal{K}}(\varsigma)(m) = \begin{cases} T_{\mathcal{F}}(\varsigma)(m); & \text{if } \varsigma \in \mathcal{A} - \mathcal{B} \\ T_{\mathcal{G}}(\varsigma)(m); & \text{if } \varsigma \in \mathcal{B} - \mathcal{A} \\ max\left(T_{\mathcal{F}}(\varsigma)(m), T_{\mathcal{G}}(e)(m)\right); & \text{if } \varsigma \in \mathcal{A} \cap \mathcal{B} \end{cases}$$

$$I_{\mathcal{K}}(\varsigma)(m) = \begin{cases} I_{\mathcal{F}}(\varsigma)(m); & \text{if } \varsigma \in \mathcal{A} - \mathcal{B} \\ I_{G}(\varsigma)(m); & \text{if } \varsigma \in \mathcal{B} - \mathcal{A} \\ min\left(I_{\mathcal{F}}(\varsigma)(m), I_{\mathcal{G}}(\varsigma)(m)\right); & \text{if } \varsigma \in \mathcal{A} \cap \mathcal{B} \end{cases}$$

$$F_{\mathcal{K}}(\varsigma)(m) = \begin{cases} F_{\mathcal{F}}(\varsigma)(m); & \text{if } \varsigma \in \mathcal{A} - \mathcal{B} \\ F_{\mathcal{G}}(\varsigma)(m); & \text{if } \varsigma \in \mathcal{B} - \mathcal{A} \\ min\left(F_{\mathcal{F}}(\varsigma)(m), F_{\mathcal{G}}(\varsigma)(m)\right); & \text{if } \varsigma \in \mathcal{A} \cap \mathcal{B} \end{cases}$$

- **Definition 6 [16]**

Consider two NSSs (F, A) and (G, B) defined over the common universe U. The intersection of (F, A) and (G, B) is represented as $(F, A) \cap (G, B)$ and is defined as $(F, A) \cap (G, B)=(H, C)$, where $C=A \cup B$. The truth-membership, indeterminacy-membership, and falsity-membership of (K, C) are determined as follows:

$$T_{\mathcal{H}}(\varsigma)(m) = \begin{cases} T_{\mathcal{F}}(\varsigma)(m); & \text{if } \varsigma \in \mathcal{A} - \mathcal{B} \\ T_{\mathcal{G}}(\varsigma)(m); & \text{if } \varsigma \in \mathcal{B} - \mathcal{A} \\ min\left(T_{\mathcal{F}}(\varsigma)(m), T_{\mathcal{G}}(e)(m)\right); & \text{if } \varsigma \in \mathcal{A} \cap \mathcal{B} \end{cases}$$

$$I_{\mathcal{H}}(\varsigma)(m)=\begin{cases}I_{\mathcal{F}}(\varsigma)(m); & \text{if } \varsigma\in\mathcal{A}-\mathcal{B}\\ I_{G}(\varsigma)(m); & \text{if } \varsigma\in\mathcal{B}-\mathcal{A}\\ max\left(I_{\mathcal{F}}(\varsigma)(m),I_{\mathcal{G}}(\varsigma)(m)\right); & \text{if } \varsigma\in\mathcal{A}\cap\mathcal{B}\end{cases}$$

$$F_{\mathcal{H}}(\varsigma)(m)=\begin{cases}F_{\mathcal{F}}(\varsigma)(m); & \text{if } \varsigma\in\mathcal{A}-\mathcal{B}\\ F_{\mathcal{G}}(\varsigma)(m); & \text{if } \varsigma\in\mathcal{B}-\mathcal{A}\\ \max\left(F_{\mathcal{F}}(\varsigma)(m),F_{\mathcal{G}}(\varsigma)(m)\right); & \text{if } \varsigma\in\mathcal{A}\cap\mathcal{B}\end{cases}$$

- **Definition 7 [4]**

An effective set denoted as Λ within a universe of discourse A, characterized by the function Λ: $A\longrightarrow[0, 1]$. The set A comprises effective parameters capable of modifying membership values, exerting a positive impact (or no impact) on these values upon application, and given by: $\Lambda=\{\langle a, \delta_\Lambda(a): a\in A\rangle\}$ where $\delta_\Lambda(a)$ is a membership degree.

- **Definition 8 [4]**

Let U be initial universal set, E be a set of parameters, A a set of effective parameters, and Λ be an effective set over A. Assume that I^U represent the collection of all fuzzy subsets of U. Then $(F, E)_\Lambda$ is an Effective Fuzzy Soft Set (EFSS) over U, and F is a mapping defined as follows: F: $E\longrightarrow I^U$ This is outlined by:

$$F(e_i)_\Lambda=\left\{\frac{x_j}{\mu_U(x_j)_\Lambda}: x_j\in U, e_i\in E\right\}, \text{where } \forall a_k\in A,$$

$$\mu_U(x_j)_\Lambda=\begin{cases}\mu_U(x_j)+\left[\frac{\left(1-\mu_U(x_j)\right)\sum_k\delta_{\Lambda_{x_j}}(a_k)}{|A|}\right], & if\ \mu_U(x_j)\in(0,1)\\ \mu_U(x_j) & O.W\end{cases}$$

- **Definition 9 [4]**

The complement of the effective set Λ with respect to the set of effective parameters A is represented as Λ^c, where the symbol c indicates the fuzzy complement operation.

- **Definition 10 [4]**

Consider $(F, E)_\Lambda$ EFSS. Then Λ complement of $(F, E)_\Lambda$ is also EFSS denoted as $(F, E)_\Lambda{}^c$, and Λ^c is fuzzy complement of Λ. In this process, remain the fuzzy soft set F unchange, and obtain the fuzzy complement Λ^c. Finally apply definition 8 to generate a new EFSS.

- **Definition 11 [4]**

Consider $(F, E)_\Lambda$ EFSS. Then $Soft_{complement}$ of $(F, E)_\Lambda$ is also EFSS and denoted as $(F^c, E)_\Lambda$. Here, F^c represents the fuzzy soft complement of F.

In this process, the effective set Λ remains unchanged, and the complement of fuzzy soft set F is obtained. Definition 8 is then applied to generate a new EFSS.

- **Definition 12 [4]**

Assume that $(F,E_1)_{\Lambda_1}$ and $(G,E_2)_{\Lambda_2}$ be EFSSs over the universe U. Then union of these sets is also an EFSS $(\mathcal{H},E)_{\Lambda_s}$ where $E=E_1\cup E_2$ and $\forall v\in E$, is given as follows:

$$\mathcal{H}_{\Lambda_s}(v)=\begin{cases}\mathcal{F}_{\Lambda_s}(v) & \text{if } v\in E_1-E_2\\ \mathcal{G}_{\Lambda_s}(v) & \text{if } v\in E_1-E_2\\ (\mathcal{F}\cup\mathcal{G})_{\Lambda_s}(v) & \text{if } v\in E_1\cap E_2\end{cases}$$

where H represents the fuzzy soft union between F and G and s denotes any s-norm.

- **Definition 13 [4]**

Assume that $(\mathcal{F},E_1)_{\Lambda_1}$ and $(\mathcal{G},E_2)_{\Lambda_2}$ be EFSSs over the universe U. Then intersection of these sets is also a EFSS $(\mathcal{K},E)_{\Lambda_s}$ where $E=E_1\cup E_2$ and $\forall v\in E$, is given as follows:

$$\mathcal{K}_{\Lambda_t}(v)=\begin{cases}\mathcal{F}_{\Lambda_t}(v) & \text{if } v\in E_1-E_2\\ \mathcal{G}_{\Lambda_t}(v) & \text{if } v\in E_1-E_2\\ (\mathcal{F}\cap\mathcal{G})_{\Lambda_t}(v) & \text{if } v\in E_1\cap E_2\end{cases}$$

where K represents the fuzzy soft union between F and G and t denotes any t-norm.

3. Effective Neutrosophic Soft Set (ENSS)

In this section, we present the fundamental definition of Effective Neutrosophic Soft Set (ENSS), accompanied by examples and properties. Assume that U be the initial universal set and $N(U)$ be the set of all neutrosophic subsets of U. Let E be the set of parameters.

- **Definition 14**

Define A as effective parameters set, and Λ as an effective set over A. Then $(\psi, E)_\Lambda$ is said to be ENSS over U, and ψ is a mapping defined as ψ: $E\rightarrow N(U)$ and is given by:

$$\psi(\mathrm{e})(x_j)_\Lambda=\left\{\frac{x_j}{\langle T_U(x_j)_\Lambda, I_U(x_j)_\Lambda, F_U(x_j)_\Lambda\rangle}: x_j\epsilon U, \mathrm{e}\in E\right\}$$

where,

$$T_U(x_j)_\Lambda=\begin{cases}T_U(x_j)+\left[\frac{[1-T_U(x_j)]\sum_k\delta_{\Lambda x_j}(a_k)}{|A|}\right] & if\ T_U(x_j)\in[0,1] I_U(x_j)_\Lambda=I(x_j)\\ T_U(x_j) & O.W\end{cases}$$

$$F_U(x_j)_\Lambda=\begin{cases}F_U(x_j)-\left[\frac{[F_U(x_j)]\sum_k\delta_{\Lambda x_j}(a_k)}{|A|}\right] & if\ F_U(x_j)\in[0,1]\\ F_U(x_j) & O.W\end{cases}$$

- **Example 1**

Consider the universe set $U=\{x_1, x_2, x_3\}$. Consider the parameters set $E=\{e_1, e_2, e_3, e_4, e_5, e_6\}$ and the effective parameters set $A=\{a_1, a_2, a_3, a_4\}$. Assume that the expert provides the effective set over A as follows:

$$\Lambda(x_1)=\left\{\frac{a_1}{0.3},\frac{a_2}{0},\frac{a_3}{1},\frac{a_4}{0.7}\right\}, \Lambda(x_2)=\left\{\frac{a_1}{0.4},\frac{a_2}{0.5},\frac{a_3}{1},\frac{a_4}{1}\right\},$$
$$\Lambda(x_3)=\left\{\frac{a_1}{0.7},\frac{a_2}{0},\frac{a_3}{0.6},\frac{a_4}{0.4}\right\}$$

Consider the NSS (ψ, E) given as follows:

$$(\psi,E)=\left\{\left(e_1,\frac{x_1}{\langle 0.5,0.2,0.4\rangle},\frac{x_2}{\langle 0.3,0.1,0.5\rangle},\frac{x_3}{\langle 0.3,0.3,0.5\rangle}\right),\right.$$
$$\left(e_2,\frac{x_1}{\langle 0.2,0.4,0.7\rangle},\frac{x_2}{\langle 0.3,0.6,0.8\rangle},\frac{x_3}{\langle 0.1,0.4,0.6\rangle}\right),$$

$$\left(e_3,\frac{x_1}{\langle 0.6,0.9,0.3\rangle},\frac{x_2}{\langle 0.4,0.7,0.1\rangle},\frac{x_3}{\langle 0.8,0.1,0.2\rangle}\right),$$
$$\left(e_4,\frac{x_1}{\langle 0.4,0.4,0.8\rangle},\frac{x_2}{\langle 0.2,0.3,0.9\rangle},\frac{x_3}{\langle 0.8,0.6,0.2\rangle}\right),$$
$$\left(e_5,\frac{x_1}{\langle 0.3,0.6,0.5\rangle},\frac{x_2}{\langle 0.5,0.3,0.2\rangle},\frac{x_3}{\langle 0.8,0.5,0.4\rangle}\right),$$
$$\left(e_6,\frac{x_1}{\langle 0.7,0.6,0.5\rangle},\frac{x_2}{\langle 0.6,0.5,0.7\rangle},\frac{x_3}{\langle 0.2,0.1,0.3\rangle}\right)\Bigg\}$$

Now, we utilize definition 14 to determine $\psi(e_1)(x_j)_\Lambda$, j = 1,2,3.

$$\psi(e_1)(x_j)_\Lambda = \begin{cases} \frac{x_1}{\langle 0.5+[1-0.5\,(0.3+0+1+0.7)/4],\ 0.2,\ 0.4-[0.4\,(0.3+0+1+0.7)/4]\rangle}, \\ \frac{x_2}{\langle 0.3+[1-0.3\,(0.4+0.5+1+1)/4],\ 0.1,\ 0.5-[0.5\,(0.4+0.5+1+1)/4]\rangle}, \\ \frac{x_3}{\langle 0.3+[1-0.3\,(0.7+0+0.6+0.4)/4],\ 0.3,\ 0.5-[0.5\,(0.7+0+0.6+0.4)/4]\rangle}, \end{cases}$$
$$=\left\{\frac{x_1}{\langle 0.75,0.2,0.2\rangle},\frac{x_2}{\langle 0.81,0.1,0.1\rangle},\frac{x_3}{\langle 0.6,0.3,0.29\rangle}\right\}$$

In a similar manner, the ensuing ENSS $(\psi, E)_\Lambda$ is derived as follows:

$$(\psi,E)_\Lambda=\left\{\left(e_1,\langle\frac{x_1}{\langle 0.75,0.2,0.2\rangle},\frac{x_2}{\langle 0.81,0.1,0.1\rangle},\frac{x_3}{\langle 0.6,0.3,0.29\rangle}\rangle\right),\right.$$
$$\left(e_2,\langle\frac{x_1}{\langle 0.6,0.4,0.35\rangle},\frac{x_2}{\langle 0.81,0.6,0.22\rangle},\frac{x_3}{\langle 0.48,0.4,0.35\rangle}\rangle\right),$$
$$\left(e_4,\langle\frac{x_1}{\langle 0.7,0.4,0.4\rangle},\frac{x_2}{\langle 0.78,0.3,0.25\rangle},\frac{x_3}{\langle 0.89,0.6,0.12\rangle}\rangle\right),$$
$$\left(e_5,\langle\frac{x_1}{\langle 0.65,0.6,0.25\rangle},\frac{x_2}{\langle 0.86,0.3,0.06\rangle},\frac{x_3}{\langle 0.89,0.5,0.23\rangle}\rangle\right)$$
$$\left.\left(e_6,\langle\frac{x_1}{\langle 0.85,0.6,0.25\rangle},\frac{x_2}{\langle 0.89,0.5,0.2\rangle},\frac{x_3}{\langle 0.54,0.1,0.17\rangle}\rangle\right)\right\}$$

- **Definition 15**

Assume that $(\psi,E_1)_{\Lambda_1}$ and $(\phi,E_2)_{\Lambda_2}$ are ENSSs over the universe *U*. Then $(\psi,E_1)_{\Lambda_1}$ is termed an effective neutrosophic soft subset of $(\phi,E_2)_{\Lambda_2}$ if the following holds:

1) $E_1 \subset E_2$.
2) $\Lambda_1(x) \le \Lambda_2(x)$.
3) $T_{\psi_{\Lambda_1}(e)}(x) \le T_{\phi_{\Lambda_2}(e)}(x)$, $I_{\psi_{\Lambda_1}(e)}(x) \le I_{\phi_{\Lambda_2}(e)}(x)$, $F_{\psi_{\Lambda_1}(e)}(x) \ge F_{\phi_{\Lambda_2}(e)}(x)$.

$\forall e \in E_1$, $x \in U$. We denote it by ($\forall e \in E_1, x \in U$. We denote it by $(\psi,E_1)_{\Lambda_1} \subseteq (\phi,E_2)_{\Lambda_2}$.

- **Example 2**

Let $E_1=\{e_1, e_2, e_3\}$ & $E_2=\{e_1, e_2, e_3, e_4\}$, over the common universe $U=\{x_1, x_2, x_3, x_4\}$. Consider effective sets given as follows:

$$\Lambda_1(x_1)=\left\{\frac{a_1}{0.1},\frac{a_2}{0},\frac{a_3}{0.1},\frac{a_4}{0.2}\right\},\Lambda_1(x_2)=\left\{\frac{a_1}{0.5},\frac{a_2}{0.5},\frac{a_3}{0.3},\frac{a_4}{0.2}\right\},$$
$$\Lambda_1(x_3)=\left\{\frac{a_1}{0.1},\frac{a_2}{0.1},\frac{a_3}{0.6},\frac{a_4}{0.1}\right\},\Lambda_1(x_4)=\left\{\frac{a_1}{0.1},\frac{a_2}{0.7},\frac{a_3}{0.3},\frac{a_4}{0.8}\right\}$$
$$\Lambda_2(x_1)=\left\{\frac{a_1}{0.8},\frac{a_2}{1},\frac{a_3}{0.1},\frac{a_4}{0.3}\right\},\Lambda_2(x_2)=\left\{\frac{a_1}{1},\frac{a_2}{1},\frac{a_3}{0.7},\frac{a_4}{0.6}\right\},$$
$$\Lambda_2(x_3)=\left\{\frac{a_1}{0.3},\frac{a_2}{0.2},\frac{a_3}{0.8},\frac{a_4}{0.1}\right\},\Lambda_2(x_4)=\left\{\frac{a_1}{0.2},\frac{a_2}{0.8},\frac{a_3}{0.3},\frac{a_4}{0.9}\right\}$$

Consider NSSs given as follows:

$$(\psi,E_1)=\left\{\left(e_1,\langle\frac{x_1}{\langle 0.4,0.1,0.7\rangle},\frac{x_2}{\langle 0.2,0.1,0.6\rangle},\frac{x_3}{\langle 0.1,0.1,0.7\rangle},\frac{x_4}{\langle 0.3,0.5,0.6\rangle}\rangle\right),\right.$$
$$\left(e_2,\langle\frac{x_1}{\langle 0.3,0.1,0.8\rangle},\frac{x_2}{\langle 0.4,0.1,0.6\rangle},\frac{x_3}{\langle 0.3,0.1,0.7\rangle},\frac{x_4}{\langle 0.3,0.2,0.7\rangle}\rangle\right),$$
$$\left.\left(e_3,\langle\frac{x_1}{\langle 0.4,0.2,0.9\rangle},\frac{x_2}{\langle 0.3,0.1,0.8\rangle},\frac{x_3}{\langle 0.4,0.1,0.8\rangle},\frac{x_4}{\langle 0.4,0.3,0.8\rangle}\rangle\right)\right\}$$
$$(\phi,E_2)=\left\{\left(e_1,\langle\frac{x_1}{\langle 0.5,0.2,0.6\rangle},\frac{x_2}{\langle 0.3,0.4,0.5\rangle},\frac{x_3}{\langle 0.2,0.1,0.5\rangle},\frac{x_4}{\langle 0.4,0.6,0.2\rangle}\rangle\right),\right.$$
$$\left(e_2,\langle\frac{x_1}{\langle 0.7,0.2,0.6\rangle},\frac{x_2}{\langle 0.8,0.3,0.4\rangle},\frac{x_3}{\langle 0.7,0.2,0.2\rangle},\frac{x_4}{\langle 0.8,0.2,0.1\rangle}\rangle\right),$$
$$\left(e_3,\langle\frac{x_1}{\langle 0.6,0.3,0.8\rangle},\frac{x_2}{\langle 0.5,0.2,0.6\rangle},\frac{x_3}{\langle 0.7,0.3,0.4\rangle},\frac{x_4}{\langle 0.8,0.5,0.6\rangle}\rangle\right),$$
$$\left.\left(e_4,\langle\frac{x_1}{\langle 0.7,0.2,0.6\rangle},\frac{x_2}{\langle 0.6,0.3,0.7\rangle},\frac{x_3}{\langle 0.7,0.5,0.6\rangle},\frac{x_4}{\langle 0.6,0.8,0.3\rangle}\rangle\right)\right\}$$

Then the ENSSs is given as follows:

$$(\psi,E_1)_{\Lambda_1}=\left\{\left(e_1,\langle\frac{x_1}{\langle 0.46,0.1,0.63\rangle},\frac{x_2}{\langle 0.5,0.1,0.38\rangle},\frac{x_3}{\langle 0.30,0.1,0.54\rangle},\frac{x_4}{\langle 0.63,0.5,0.32\rangle}\rangle\right)\right.$$
$$\left(e_2,\langle\frac{x_1}{\langle 0.37,0.1,0.72\rangle},\frac{x_2}{\langle 0.63,0.1,0.38\rangle},\frac{x_3}{\langle 0.46,0.1,0.54\rangle},\frac{x_4}{\langle 0.63,0.2,0.37\rangle}\rangle\right),$$
$$\left.\left(e_3,\langle\frac{x_1}{\langle 0.46,0.2,0.81\rangle},\frac{x_2}{\langle 0.56,0.1,0.5\rangle},\frac{x_3}{\langle 0.54,0.1,0.62\rangle},\frac{x_4}{\langle 0.69,0.3,0.42\rangle}\rangle\right)\right\}$$
$$(\phi,E_2)_{\Lambda_2}=\left\{\left(e_1,\langle\frac{x_1}{\langle 0.78,0.2,0.27\rangle},\frac{x_2}{\langle 0.88,0.4,0.09\rangle},\frac{x_3}{\langle 0.48,0.1,0.32\rangle},\frac{x_4}{\langle 0.73,0.6,0.09\rangle}\rangle\right)\right.$$
$$\left(e_2,\langle\frac{x_1}{\langle 0.87,0.2,0.27\rangle},\frac{x_2}{\langle 0.97,0.3,0.07\rangle},\frac{x_3}{\langle 0.81,0.2,0.13\rangle},\frac{x_4}{\langle 0.91,0.2,0.05\rangle}\rangle\right),$$
$$\left(e_3,\langle\frac{x_1}{\langle 0.82,0.3,0.36\rangle},\frac{x_2}{\langle 0.91,0.2,0.11\rangle},\frac{x_3}{\langle 0.81,0.3,0.26\rangle},\frac{x_4}{\langle 0.91,0.5,0.27\rangle}\rangle\right),$$
$$\left.\left(e_4,\langle\frac{x_1}{\langle 0.87,0.2,0.27\rangle},\frac{x_2}{\langle 0.93,0.3,0.12\rangle},\frac{x_3}{\langle 0.81,0.5,0.39\rangle},\frac{x_4}{\langle 0.82,0.8,0.14\rangle}\rangle\right)\right\}$$

It's clear $E_1 \subset E_2, \Lambda_1 \subset \Lambda_2$ *and* $T_{\psi_{\Lambda_1}(e)} \le T_{\phi_{\Lambda_2}(e)}$, $I_{\psi_{\Lambda_1}(e)} \le I_{\phi_{\Lambda_2}(e)}$, $F_{\psi_{\Lambda_1}(e)} \ge F_{\phi_{\Lambda_2}(e)}$ $\forall e \in E_1$. Then $(\psi,E_1)_{\Lambda_1} \subset (\phi,E_2)_{\Lambda_2}$.

- **Definition 16**

Assume that $(\psi,E_1)_{\Lambda_1}$ and $(\phi,E_2)_{\Lambda_2}$ are ENSSs over *U*. Then $(\psi,E_1)_{\Lambda_1}$ is equal to $(\phi,E_2)_{\Lambda_2}$ and denoted by $(\psi,E_1)_{\Lambda_1}=(\phi,E_2)_{\Lambda_2}$ if $(\psi,E_1)_{\Lambda_1}$ is an ENS subset of $(\phi,E_2)_{\Lambda_2}$ and $(\phi,E_2)_{\Lambda_2}$ is an ENS subset of $(\psi,E_1)_{\Lambda_1}$

4. Basic Operations

In this section, we introduce operations related to ENSS, specifically the subset, equal, complement, union, and intersection. We offer illustrative examples to elucidate these concepts and outline their fundamental properties.

- **Definition 17**

Consider $(\psi, E)_\Lambda$ be ENSS. Then the $Total_{complement}$ of $(\psi, E)_\Lambda$ is also ENSS and represented as $(\psi^c, E)_\Lambda{}^c$. Here ψ^c is neutrosophic complement of ψ and Λ^c stands for fuzzy complement of Λ.

In this process, both the fuzzy complement Λ^c and the complement of NSS ψ^c are obtained. Definition 14 is then applied to generate a new ENSS.

- **Definition 18**

Consider $(\psi, E)_\Lambda$ be ENSS. Then the $\Lambda_{complement}$ of $(\psi, E)_\Lambda$ is also ENSS represented as $(\psi, E)_\Lambda{}^c$, here Λ^c stands for any fuzzy complement of Λ. In this process, the NSS ψ remains unchanged, and the fuzzy complement Λ^c is obtained. Definition 14 is then applied to generate a new ENSS.

- **Definition 19**

Consider $(\psi, E)_\Lambda$ be ENSS. Then the *Soft*$_{complement}$ of $(\psi, E)_\Lambda$ is also ENSS and represented as $(\psi^c, E)_\Lambda$, here ψ^c is neutrosophic soft complement of ψ. In this process, the effective set Λ remains unchanged, and the complement of NSS ψ is obtained. Definition 14 is then applied to generate a new ENSS.

- **Example 3**

Let $E=\{e_1, e_2, e_3\}$ over the common universe $U=\{x_1, x_2, x_3, x_4\}$. Consider effective sets given as follows:

$$\Lambda(x_1)=\left\{\frac{a_1}{0.3},\frac{a_2}{0},\frac{a_3}{1},\frac{a_4}{0.7}\right\},\ \Lambda(x_2)=\left\{\frac{a_1}{0.4},\frac{a_2}{0.5},\frac{a_3}{1},\frac{a_4}{1}\right\},$$
$$\Lambda(x_3)=\left\{\frac{a_1}{0.7},\frac{a_2}{0},\frac{a_3}{0.6},\frac{a_4}{0.4}\right\}$$

Consider the NSS (ψ, E) given as follows:

$$(\psi,E)=\left\{\left(e_1,\frac{x_1}{\langle 0.5,0.2,0.4\rangle},\frac{x_2}{\langle 0.3,0.1,0.5\rangle},\frac{x_3}{\langle 0.3,0.3,0.5\rangle}\right),\left(e_2,\frac{x_1}{\langle 0.2,0.4,0.7\rangle},\frac{x_2}{\langle 0.3,0.6,0.8\rangle},\frac{x_3}{\langle 0.1,0.4,0.6\rangle}\right),\left(e_3,\frac{x_1}{\langle 0.6,0.9,0.3\rangle},\frac{x_2}{\langle 0.4,0.7,0.1\rangle},\frac{x_3}{\langle 0.8,0.1,0.2\rangle}\right)\right\}$$

Now, we obtain the complement of effective set Λ^c as follows:

$$(\psi,E)^c=\left\{\left(e_1,\frac{x_1}{\langle 0.4,0.2,0.5\rangle},\frac{x_2}{\langle 0.5,0.1,0.3\rangle},\frac{x_3}{\langle 0.5,0.3,0.3\rangle}\right),\left(e_2,\frac{x_1}{\langle 0.7,0.4,0.2\rangle},\frac{x_2}{\langle 0.8,0.6,0.3\rangle},\frac{x_3}{\langle 0.6,0.4,0.1\rangle}\right),\left(e_3,\frac{x_1}{\langle 0.3,0.9,0.6\rangle},\frac{x_2}{\langle 0.10.7,0.4\rangle},\frac{x_3}{\langle 0.2,0.1,0.8\rangle}\right)\right\}$$

Now, we apply definitions19, 17, 18, and 14 to find *Total*$_{complement}$, $\Lambda_{complement}$ and *Soft*$_{complement}$ respectively.
To find *Total*$_{complement}$, first we find $\psi^c(e_1)(x_j)_{\Lambda^c}, j=1,2,3$, as follows:

$$\psi^c(e_1)(x_j)_{\Lambda^c}=\left\{\begin{array}{l}\frac{x_1}{\langle 0.4+[1-0.4\,(0.7+1+0+0.3)/4],\ 0.2,\ 0.5-[0.5\,(0.7+1+0+0.3)/4]\rangle},\\ \frac{x_2}{\langle 0.5+[1-0.5\,(0.6+0.5+0+0)/4],\ 0.1,\ 0.3-[0.3\,(0.6+0.5+0+0)/4]\rangle},\\ \frac{x_3}{\langle 0.5+[1-0.5\,(0.3+1+0.4+0.6)/4],\ 0.3,\ 0.3-[0.3\,(0.3+1+0.4+0.6)/4]\rangle}\end{array}\right.$$
$$=\left\{\frac{x_1}{\langle 0.7,0.2,0.25\rangle},\frac{x_2}{\langle 0.64,0.1,0.22\rangle},\frac{x_3}{\langle 0.79,0.3,0.13\rangle}\right\}$$

In a similar manner, we find *Total*$_{complement}$$=(\psi,E)^c{}_{\Lambda^c}$ as shown below:

$$(\psi,E)^c{}_{\Lambda^c}=\left\{\left(e_1,\left\langle\frac{x_1}{\langle 0.7,0.2,0.25\rangle},\frac{x_2}{\langle 0.64,0.1,0.22\rangle},\frac{x_3}{\langle 0.79,0.3,0.13\rangle}\right\rangle\right),\left(e_2,\left\langle\frac{x_1}{\langle 0.85,0.4,0.1\rangle},\frac{x_2}{\langle 0.86,0.6,0.22\rangle},\frac{x_3}{\langle 0.83,0.4,0.04\rangle}\right\rangle\right),\left(e_3,\left\langle\frac{x_1}{\langle 0.65,0.9,0.3\rangle},\frac{x_2}{\langle 0.35,0.7,0.29\rangle},\frac{x_3}{\langle 0.66,0.1,0.34\rangle}\right\rangle\right)\right\}$$

To find $\Lambda_{complement}$, first we find $\psi(e_1)(x_j)_{\Lambda^c}, j=1,2,3$ as follows:

$$\psi(e_1)(x_j)_{\Lambda^c}=\left\{\begin{array}{l}\frac{x_1}{\langle 0.5+[1-0.5\,(0.7+1+0+0.3)/4],\ 0.2,\ 0.4-[0.4\,(0.7+1+0+0.3)/4]\rangle},\\ \frac{x_2}{\langle 0.3+[1-0.3\,(0.6+0.5+0+0)/4],\ 0.1,\ 0.5-[0.5\,(0.6+0.5+0+0)/4]\rangle},\\ \frac{x_3}{\langle 0.3+[1-0.3\,(0.3+1+0.4+0.6)/4],\ 0.3,\ 0.5-[0.5\,(0.3+1+0.4+0.6)/4]\rangle}\end{array}\right.$$
$$=\left\{\frac{x_1}{\langle 0.75,0.2,0.2\rangle},\frac{x_2}{\langle 0.49,0.1,0.36\rangle},\frac{x_3}{\langle 0.70,0.3,0.21\rangle}\right\}$$

In a similar manner, we find $\Lambda_{\text{complement}}=(\psi,E)_{\Lambda^c}$ as shown below:

$$(\psi,E)_{\Lambda^c}=\left\{\left(e_1,\left\langle\frac{x_1}{\langle 0.75,0.2,0.2\rangle},\frac{x_2}{\langle 0.49,0.1,0.36\rangle},\frac{x_3}{\langle 0.70,0.3,0.21\rangle}\right\rangle\right),\left(e_2,\left\langle\frac{x_1}{\langle 0.6,0.4,0.35\rangle},\frac{x_2}{\langle 0.49,0.6,0.58\rangle},\frac{x_3}{\langle 0.62,0.4,0.26\rangle}\right\rangle\right),\left(e_3,\left\langle\frac{x_1}{\langle 0.8,0.9,0.15\rangle},\frac{x_2}{\langle 0.57,0.7,0.07\rangle},\frac{x_3}{\langle 0.92,0.1,0.09\rangle}\right\rangle\right)\right\}$$

To find *Soft*$_{complement}$, first we find $\psi^c(e_1)(x_j)_\Lambda, j=1,2,3$ as follows:

$$\psi^c(e_1)(x_j)_\Lambda=\left\{\begin{array}{l}\frac{x_1}{\langle 0.4+[1-0.4\,(0.3+0+1+0.7)/4],\ 0.2,\ 0.5-[0.5\,(0.3+0+1+0.7)/4]\rangle},\\ \frac{x_2}{\langle 0.5+[1-0.5\,(0.4+0.5+1+1)/4],\ 0.1,\ 0.3-[0.3\,(0.4+0.5+1+1)/4]\rangle},\\ \frac{x_3}{\langle 0.5+[1-0.5\,(0.7+0+0.6+0.4)/4],\ 0.3,\ 0.3-[0.3\,(0.7+0+0.6+0.4)/4]\rangle}\end{array}\right.$$
$$=\left\{\frac{x_1}{\langle 0.75,0.2,0.25\rangle},\frac{x_2}{\langle 0.86,0.1,0.08\rangle},\frac{x_3}{\langle 0.29,0.3,0.17\rangle}\right\}$$

In a similar manner, we find the in a similar manner, we find the $Soft_{complement}=(\psi,E)^c_\Lambda$ as shown below:

$$(\psi,E)^c_\Lambda=\left\{\left(e_1,\left\langle\frac{x_1}{\langle 0.7,0.2,0.25\rangle},\frac{x_2}{\langle 0.86,0.1,0.08\rangle},\frac{x_3}{\langle 0.29,0.3,0.17\rangle}\right\rangle\right),\left(e_2,\left\langle\frac{x_1}{\langle 0.85,0.4,0.1\rangle},\frac{x_2}{\langle 0.95,0.6,0.08\rangle},\frac{x_3}{\langle 0.77,0.4,0.06\rangle}\right\rangle\right),\left(e_3,\left\langle\frac{x_1}{\langle 0.65,0.9,0.3\rangle},\frac{x_2}{\langle 0.75,0.7,0.11\rangle},\frac{x_3}{\langle 0.54,0.1,0.46\rangle}\right\rangle\right)\right\}$$

- **Proposition 1**

Let $(\psi, E)_\Lambda$ be ENSS over the U. Then

a) *Total*$_{complement}$ (*Total*$_{complement}$ $(\psi, E)_\Lambda$)=$(\psi, E)_\Lambda$, $\left((\psi,E)^c_{\Lambda^c}\right)^c=(\psi,E)_\Lambda)$.
b) $\Lambda_{complement}$ ($\Lambda_{complement}$ $(\psi, E)_\Lambda$)=$(\psi, E)_\Lambda$.
c) *Soft*$_{complement}$ (*Soft*$_{complement}$ $(\psi, E)_\Lambda$)=$(\psi, E)_\Lambda$.

- **Proof**

We want to prove (a) as following:

Let $(\psi, E)_\Lambda$ be ENSS over the U and $\forall\ x_j \in U$, $e\in E$

$$(\psi,E)_\Lambda=\left\{\left(e,\frac{x_j}{\langle T_U(x_j)_\Lambda, I_U(x_j)_\Lambda, F_U(x_j)_\Lambda\rangle}\right)\right\}$$

$$(\psi,E)^c_{\Lambda^c}=\left\{\left(\mathrm{e},\frac{x_j}{\langle F_{U^c}(x_j)_{\Lambda^c}, I_{U^c}(x_j)_{\Lambda^c}, T_{U^c}(x_j)_{\Lambda^c}\rangle}\right)\right\}$$

$$((\psi,E)^c_{\Lambda^c})^c=\left\{\left(\mathrm{e},\frac{x_j}{\langle \left(T_{U^c}(x_j)_{\Lambda^c}\right)^c, \left(I(x_j)_{\Lambda^c}\right)^c, \left(F_{U^c}(x_j)_{\Lambda^c}\right)^c\rangle}\right)\right\}$$

where,

$$\left(T_{U^c}(x_j)_{\Lambda^c}\right)^c=T_{(U^c)^c}(x_j)_{(\Lambda^c)^c}$$
$$=\left\{\begin{cases}T_{U^c}(x_j)+\left[\dfrac{[1-T_{U^c}(x_j)]\sum_k\delta_{\Lambda^c_{x_j}}(a_k)}{|A|}\right] & if\ \ T_U(x_j)\in[0,1]\\ T_{U^c}(x_j) & o.w\end{cases}\right\}^c$$
$$=\begin{cases}T_{(U^c)^c}(x_j)+\left[\dfrac{[1-T_{(U^c)^c}(x_j)]\sum_k\delta_{\left(\Lambda^c_{x_j}\right)^c}(a_k)}{|A|}\right] & if\ \ T_U(x_j)\in[0,1]\\ T_{(U^c)^c}(x_j) & o.w\end{cases}$$

$$= \begin{cases} T_U(x_j) + \left[\frac{[1-T_U(x_j)]\sum_k \delta_{\Lambda x_j}(a_k)}{|A|}\right] & if\ T_U(x_j) \in [0,1] = T_U(x_j)_\Lambda \\ T_U(x_j) & o.w \end{cases}$$

So, $\left(T_{U^c}(x_j)_{\Lambda^c}\right)^c = T_U(x_j)_\Lambda$. Similarly, $\left(I(x_j)_{\Lambda^c}\right)^c = I_U(x_j)_\Lambda$ and $\left(F_{U^c}(x_j)_{\Lambda^c}\right)^c = F_U(x_j)_\Lambda$. Then$((\psi,E)^c_{\Lambda^c})^c = (\psi,E)_\Lambda$.

The verification of (b) and (c) can be readily derived from their respective definitions.

- **Definition 20**

Consider two ENSSs $(\psi, E_1)_{\Lambda_1}$ and $(\phi, E_2)_{\Lambda_2}$ over the common universe *U*. Then the union of these sets also is ENSS $(\Upsilon, E)_{\Lambda_s}$ where $E = E_1 \cup E_2$ and $\forall \varsigma \in E$, is provided in the following manner:

$$\Upsilon_{\Lambda_s}(v) = \begin{cases} \psi_{\Lambda_1}(\varsigma) & \text{if } \varsigma \in E_1 - E_2 \\ \phi_{\Lambda_2}(\varsigma) & \text{if } \varsigma \in E_1 - E_2 \\ (\psi \cup \phi)_{\Lambda_s}(\varsigma) & \text{if } \varsigma \in E_1 \cap E_2 \end{cases}$$

Here Υ represents the neutrosophic soft union between ψ and ϕ, Λ_s=*max*(Λ_1, Λ_2)and *s* denotes any *s*-norm.

- **Example 4**

Consider the effective set given as follows:

$\Lambda_1(x_1) = \left\{\frac{a_1}{0.3}, \frac{a_2}{0}, \frac{a_3}{1}, \frac{a_4}{0.7}\right\}$, $\Lambda_1(x_2) = \left\{\frac{a_1}{0.4}, \frac{a_2}{0.5}, \frac{a_3}{1}, \frac{a_4}{1}\right\}$,
$\Lambda_1(x_3) = \left\{\frac{a_1}{0.7}, \frac{a_2}{0}, \frac{a_3}{0.6}, \frac{a_4}{0.4}\right\}$
$\Lambda_2(x_1) = \left\{\frac{a_1}{0.7}, \frac{a_2}{0.5}, \frac{a_3}{0.2}, \frac{a_4}{1}\right\}$, $\Lambda_2(x_2) = \left\{\frac{a_1}{0.3}, \frac{a_2}{0.7}, \frac{a_3}{0.8}, \frac{a_4}{0.4}\right\}$,
$\Lambda_2(x_3) = \left\{\frac{a_1}{0.2}, \frac{a_2}{0.5}, \frac{a_3}{0.3}, \frac{a_4}{0.9}\right\}$

Consider two NSSs given as follows:

$$(\psi, E_1) = \left\{\left(e_1, \langle\frac{x_1}{\langle 0.5,0.2,0.4\rangle}, \frac{x_2}{\langle 0.3,0.1,0.5\rangle}, \frac{x_3}{\langle 0.3,0.3,0.5\rangle}\rangle\right), \left(e_3, \langle\frac{x_1}{\langle 0.6,0.9,0.3\rangle}, \frac{x_2}{\langle 0.4,0.7,0.1\rangle}, \frac{x_3}{\langle 0.8,0.1,0.2\rangle}\rangle\right), \left(e_4, \langle\frac{x_1}{\langle 0.4,0.4,0.8\rangle}, \frac{x_2}{\langle 0.2,0.3,0.9\rangle}, \frac{x_3}{\langle 0.8,0.6,0.2\rangle}\rangle\right)\right.$$

$$(\phi, E_2) = \left\{\left(e_1, \langle\frac{x_1}{\langle 0.3,0.1,0.5\rangle}, \frac{x_2}{\langle 0.1,0,0.4\rangle}, \frac{x_3}{\langle 0.4,0.2,0.9\rangle}\rangle\right), \left(e_2, \langle\frac{x_1}{\langle 0.2,0.4,0.7\rangle}, \frac{x_2}{\langle 0.3,0.6,0.8\rangle}, \frac{x_3}{\langle 0.1,0.4,0.6\rangle}\rangle\right), \left(e_4, \langle\frac{x_1}{\langle 0.5,0.3,0.9\rangle}, \frac{x_2}{\langle 0.4,0.2,0.7\rangle}, \frac{x_3}{\langle 0.9,0.3,0.4\rangle}\rangle\right), \left(e_5, \langle\frac{x_1}{\langle 0.3,0.6,0.5\rangle}, \frac{x_2}{\langle 0.5,0.3,0.2\rangle}, \frac{x_3}{\langle 0.8,0.5,0.4\rangle}\rangle\right)\right\}$$

Now, we employ the basic fuzzy union operation to find Λ_s from Λ_1 and Λ_2 in the following manner:

$\Lambda_s(x_1) = \left\{\frac{a_1}{0.7}, \frac{a_2}{0.5}, \frac{a_3}{1}, \frac{a_4}{1}\right\}$, $\Lambda_s(x_2) = \left\{\frac{a_1}{0.4}, \frac{a_2}{0.7}, \frac{a_3}{1}, \frac{a_4}{1}\right\}$
$\Lambda_s(x_3) = \left\{\frac{a_1}{0.7}, \frac{a_2}{0.5}, \frac{a_3}{0.6}, \frac{a_4}{0.9}\right\}$

Then, the NSS union (Υ, E) givn as follows:

$$(\Upsilon, E) = \left\{\left(e_1, \langle\frac{x_1}{\langle 0.5,0.1,0.4\rangle}, \frac{x_2}{\langle 0.3,0,0.4\rangle}, \frac{x_3}{\langle 0.4,0.2,0.5\rangle}\rangle\right), \left(e_2, \langle\frac{x_1}{\langle 0.2,0.4,0.7\rangle}, \frac{x_2}{\langle 0.3,0.6,0.8\rangle}, \frac{x_3}{\langle 0.1,0.4,0.6\rangle}\rangle\right), \left(e_3, \langle\frac{x_1}{\langle 0.6,0.9,0.3\rangle}, \frac{x_2}{\langle 0.4,0.7,0.1\rangle}, \frac{x_3}{\langle 0.8,0.1,0.2\rangle}\rangle\right), \left(e_4, \langle\frac{x_1}{\langle 0.5,0.3,0.8\rangle}, \frac{x_2}{\langle 0.4,0.2,0.7\rangle}, \frac{x_3}{\langle 0.9,0.3,0.2\rangle}\rangle\right), \left(e_5, \langle\frac{x_1}{\langle 0.3,0.6,0.5\rangle}, \frac{x_2}{\langle 0.5,0.3,0.2\rangle}, \frac{x_3}{\langle 0.8,0.5,0.4\rangle}\rangle\right)\right\}$$

Then by using definitions 20 and 14 we get the following ENSS

$$(\Upsilon, E)_{\Lambda_s} = \left\{\left(e_1, \langle\frac{x_1}{\langle 0.9, 0.1, 0.08\rangle}, \frac{x_2}{\langle 0.84, 0, 0.09\rangle}, \frac{x_3}{\langle 0.81, 0.2, 0.16\rangle}\rangle\right), \left(e_2, \langle\frac{x_1}{\langle 0.84, 0.4, 0.14\rangle}, \frac{x_2}{\langle 0.84, 0.6, 0.18\rangle}, \frac{x_3}{\langle 0.71, 0.4, 0.2\rangle}\rangle\right), \left(e_3, \langle\frac{x_1}{\langle 0.92, 0.9, 0.06\rangle}, \frac{x_2}{\langle 0.87, 0.7, 0.02\rangle}, \frac{x_3}{\langle 0.94, 0.1, 0.07\rangle}\rangle\right), \left(e_4, \langle\frac{x_1}{\langle 0.9, 0.3, 0.16\rangle}, \frac{x_2}{\langle 0.87, 0.2, 0.16\rangle}, \frac{x_3}{\langle 0.97, 0.3, 0.07\rangle}\rangle\right), \left(e_5, \langle\frac{x_1}{\langle 0.86, 0.6, 0.1\rangle}, \frac{x_2}{\langle 0.89, 0.3, 0.05\rangle}, \frac{x_3}{\langle 0.94, 0.5, 0.13\rangle}\rangle\right)\right\}$$

- **Definition 21**

Consider two ENSSs $(\psi, E_1)_{\Lambda_1}$ and $(\phi, E_2)_{\Lambda_2}$ over the common universe *U*. Then the intersection of these sets also is ENSS $(\Omega, E)_{\Lambda_t}$ where E=$E_1 \cup E_2$ and $\forall \varsigma \in E$, is provided in the following manner:

$$\Omega_{\Lambda_t}(v) = \begin{cases} \psi_{\Lambda_1}(\varsigma) & \text{if } \varsigma \in E_1 - E_2 \\ \phi_{\Lambda_2}(\varsigma) & \text{if } \varsigma \in E_1 - E_2 \\ (\psi \cap \phi)_{\Lambda_s}(\varsigma) & \text{if } \varsigma \in E_1 \cap E_2 \end{cases}$$

Here Ω represents the neutrosophic soft union between ψ and ϕ, Λ_t=*min*(Λ_1, Λ_2) and *t* denotes any *t*-norm.

- **Example 5**

Consider example 4, then we employ the basic fuzzy intersection operation to find Λ_t from Λ_1 and Λ_2 in the following manner:

$\Lambda_t(x_1) = \left\{\frac{a_1}{0.3}, \frac{a_2}{0}, \frac{a_3}{0.2}, \frac{a_4}{0.7}\right\}$, $\Lambda_t(x_2) = \left\{\frac{a_1}{0.3}, \frac{a_2}{0.5}, \frac{a_3}{0.8}, \frac{a_4}{0.4}\right\}$,
$\Lambda_t(x_3) = \left\{\frac{a_1}{0.2}, \frac{a_2}{0}, \frac{a_3}{0.3}, \frac{a_4}{0.4}\right\}$

Then, the NSS intersection (Ω, *E*) given as follows:

$$(\Omega, E) = \left\{\left(e_1, \langle\frac{x_1}{\langle 0.3,0.2,0.5\rangle}, \frac{x_2}{\langle 0.1,0.1,0.5\rangle}, \frac{x_3}{\langle 0.3,0.3,0.9\rangle}\rangle\right), \left(e_2, \langle\frac{x_1}{\langle 0.2,0.4,0.7\rangle}, \frac{x_2}{\langle 0.3,0.6,0.8\rangle}, \frac{x_3}{\langle 0.1,0.4,0.6\rangle}\rangle\right), \left(e_3, \langle\frac{x_1}{\langle 0.6,0.9,0.3\rangle}, \frac{x_2}{\langle 0.4,0.7,0.1\rangle}, \frac{x_3}{\langle 0.8,0.1,0.2\rangle}\rangle\right), \left(e_4, \langle\frac{x_1}{\langle 0.4,0.4,0.9\rangle}, \frac{x_2}{\langle 0.2,0.3,0.9\rangle}, \frac{x_3}{\langle 0.8,0.6,0.4\rangle}\rangle\right), \left(e_5, \langle\frac{x_1}{\langle 0.3,0.6,0.5\rangle}, \frac{x_2}{\langle 0.5,0.3,0.2\rangle}, \frac{x_3}{\langle 0.8,0.5,0.4\rangle}\rangle\right)\right\}$$

Now, by using definitions 21 and 14, we get ENSS $(\Omega, E)_{\Lambda_t}$as follows:

$$(\Omega, E)_{\Lambda_t} = \left\{\left(e_1, \langle\frac{x_1}{\langle 0.51, 0.2, 0.35\rangle}, \frac{x_2}{\langle 0.55, 0.1, 0.25\rangle}, \frac{x_3}{\langle 0.46, 0.3, 0.4\rangle}\rangle\right), \left(e_2, \langle\frac{x_1}{\langle 0.44, 0.4, 0.49\rangle}, \frac{x_2}{\langle 0.65, 0.6, 0.4\rangle}, \frac{x_3}{\langle 0.30, 0.4, 0.47\rangle}\rangle\right), \left(e_3, \langle\frac{x_1}{\langle 0.72, 0.9, 0.21\rangle}, \frac{x_2}{\langle 0.7,0.7,0.05\rangle}, \frac{x_3}{\langle 0.85, 0.1, 0.16\rangle}\rangle\right), \left(e_4, \langle\frac{x_1}{\langle 0.58, 0.4, 0.63\rangle}, \frac{x_2}{\langle 0.6,0.3,0.45\rangle}, \frac{x_3}{\langle 0.85,0.6,0.31\rangle}\rangle\right), \left(e_5, \langle\frac{x_1}{\langle 0.51, 0.6, 0.35\rangle}, \frac{x_2}{\langle 0.75, 0.3, 0.1\rangle}, \frac{x_3}{\langle 0.85, 0.5, 0.31\rangle}\rangle\right)\right\}$$

- **Proposition 2**

Let $(\psi, E_1)_{\Lambda_1}, (\phi, E_2)_{\Lambda_2}$ and $(\sigma, E_3)_{\Lambda_3}$ be three ENSSs over the common universe *U*. Then,

1) $(\psi, E_1)_{\Lambda_1} \cup (\phi, E_2)_{\Lambda_2} = (\phi, E_2)_{\Lambda_2} \cup (\psi, E_1)_{\Lambda_1}$
2) $(\psi, E_1)_{\Lambda_1} \cap (\phi, E_2)_{\Lambda_2} = (\phi, E_2)_{\Lambda_2} \cap (\psi, E_1)_{\Lambda_1}$

- **Proof**

1) Let $\forall v \in E$. In the subsequent proof, the first two cases are straightforward; therefore, we focus solely on the third case.

$$(\psi, E_1)_{\Lambda_1} \cup (\phi, E_2)_{\Lambda_2} = \left\{e, \langle v/\left(max\left(T_{\psi_{\Lambda_1}}(v), T_{\phi_{\Lambda_2}}(v)\right)\right), \left(min\left(I_{\psi_{\Lambda_1}}(v), I_{\phi_{\Lambda_2}}(v)\right)\right), \left(min\left(F_{\psi_{\Lambda_1}}(v), F_{\phi_{\Lambda_2}}(v)\right)\right)\rangle, \Lambda_s(v) = max(\Lambda_1(v), \Lambda_2(v))\right\}$$

$$= \left\{e, \langle v/\left(max\left(T_{\phi_{\Lambda_2}}(v), T_{\psi_{\Lambda_1}}(v)\right)\right), \left(min\left(I_{\phi_{\Lambda_2}}(v), I_{\psi_{\Lambda_1}}(v)\right)\right), \left(min\left(F_{\phi_{\Lambda_2}}(v), F_{\psi_{\Lambda_1}}(v)\right)\right)\rangle, \Lambda_s(v) = max(\Lambda_2(v), \Lambda_1(v))\right\}$$

$$= (\phi, E_2)_{\Lambda_2} \cup (\psi, E_1)_{\Lambda_1}$$

2) The proof is straightforward.

5. AND and OR Operations

Within this section, we define the AND and OR operations and scrutinize their individual properties.

- **Definition 22**

Consider two ENSSs $(\psi, \mathrm{E}_1)_{\Lambda_1}$ and $(\phi, \mathrm{E}_2)_{\Lambda_2}$ over the universe *U*. AND operation between these sets, written as $(\psi, \mathrm{E}_1)_{\Lambda_1} \wedge (\phi, \mathrm{E}_2)_{\Lambda_2}$, is defined as

$$(\psi, \mathrm{E}_1)_{\Lambda_1} \wedge (\phi, \mathrm{E}_2)_{\Lambda_2} = (\Theta, \mathrm{E}_1 \times \mathrm{E}_2)_{\Lambda_t}$$

where $\Theta_{\Lambda_t}(a, b) = (\psi(a) \cap \phi(b))_{\Lambda_t} : \forall (a, b) \in \mathrm{E}_1 \times \mathrm{E}_2$ $\cap$ is effective neutrosophic soft intersection and *t* represents any *t*-norm.

- **Example 6**

Consider the effective sets given as follows:

$$\Lambda_1(x_1) = \left\{\frac{a_1}{0.3}, \frac{a_2}{0}, \frac{a_3}{1}, \frac{a_4}{0.7}\right\}, \Lambda_1(x_2) = \left\{\frac{a_1}{0.4}, \frac{a_2}{0.5}, \frac{a_3}{1}, \frac{a_4}{1}\right\}$$
$$\Lambda_1(x_3) = \left\{\frac{a_1}{0.7}, \frac{a_2}{0}, \frac{a_3}{0.6}, \frac{a_4}{0.4}\right\}$$

$$\Lambda_2(x_1) = \left\{\frac{a_1}{0.7}, \frac{a_2}{0.5}, \frac{a_3}{0.2}, \frac{a_4}{1}\right\}, \Lambda_2(x_2) = \left\{\frac{a_1}{0.3}, \frac{a_2}{0.7}, \frac{a_3}{0.8}, \frac{a_4}{0.4}\right\}$$
$$\Lambda_2(x_3) = \left\{\frac{a_1}{0.2}, \frac{a_2}{0.5}, \frac{a_3}{0.3}, \frac{a_4}{0.9}\right\}$$

Consider two NSSs given as follows:

$$(\psi, E_1) = \left\{\left(e_1, \langle\frac{x_1}{\langle 0.5,0.2,0.4\rangle}, \frac{x_2}{\langle 0.3,0.1,0.5\rangle}, \frac{x_3}{\langle 0.3,0.3,0.5\rangle}\rangle\right), \left(e_3, \langle\frac{x_1}{\langle 0.6,0.9,0.3\rangle}, \frac{x_2}{\langle 0.4,0.7,0.1\rangle}, \frac{x_3}{\langle 0.8,0.1,0.2\rangle}\rangle\right)\right\}$$

$$(\phi, E_2) = \left\{\left(e_1, \langle\frac{x_1}{\langle 0.3,0.1,0.5\rangle}, \frac{x_2}{\langle 0.1,0,0.4\rangle}, \frac{x_3}{\langle 0.4,0.2,0.9\rangle}\rangle\right), \left(e_2, \langle\frac{x_1}{\langle 0.2,0.4,0.7\rangle}, \frac{x_2}{\langle 0.3,0.6,0.8\rangle}, \frac{x_3}{\langle 0.1,0.4,0.6\rangle}\rangle\right)\right\}$$

We employ the basic fuzzy intersection operation to find Λ_t from Λ_1 and Λ_2 in the following manner:

$$\Lambda_t(x_1) = \left\{\frac{a_1}{0.3}, \frac{a_2}{0}, \frac{a_3}{0.2}, \frac{a_4}{0.7}\right\}, \Lambda_t(x_2) = \left\{\frac{a_1}{0.3}, \frac{a_2}{0.5}, \frac{a_3}{0.8}, \frac{a_4}{0.4}\right\},$$
$$\Lambda_t(x_3) = \left\{\frac{a_1}{0.2}, \frac{a_2}{0}, \frac{a_3}{0.3}, \frac{a_4}{0.4}\right\}$$

Through the utilization of the neutrosophic soft intersection, we obtain $(\psi, E_1) \wedge (\phi, E_2) = (\Theta, E_1 \times E_2)$ as follows:

$$(\Theta, \mathrm{E}_1 \times \mathrm{E}_2) = \left\{\left((e_1, e_1), \frac{x_1}{\langle 0.3, 0.2, 0.5\rangle}, \frac{x_2}{\langle 0.1, 0.1, 0.5\rangle}, \frac{x_3}{\langle 0.3, 0.3, 0.9\rangle}\right), \left((e_1, e_2), \frac{x_1}{\langle 0.2, 0.4, 0.7\rangle}, \frac{x_2}{\langle 0.3, 0.6, 0.8\rangle}, \frac{x_3}{\langle 0.1, 0.4, 0.6\rangle}\right), \left((e_3, e_1), \frac{x_1}{\langle 0.3, 0.9, 0.5\rangle}, \frac{x_2}{\langle 0.1, 0.7, 0.4\rangle}, \frac{x_3}{\langle 0.4, 0.2, 0.9\rangle}\right) \left((e_3, e_2), \frac{x_1}{\langle 0.2,0.9, 0.7\rangle}, \frac{x_2}{\langle 0.3, 0.7, 0.8\rangle}, \frac{x_3}{\langle 0.1, 0.4, 0.6\rangle}\right)\right\}$$

Then, by using definitions 22 and 14 we get the ENSES $(\Theta, E_1 \times E_2)_{\Lambda_t}$ as follows:

$$(\Theta, \mathrm{E}_1 \times \mathrm{E}_2)_{\Lambda_t} = \left\{\left((e_1, e_1), \frac{x_1}{\langle 0.51, 0.2, 0.35\rangle}, \frac{x_2}{\langle 0.55, 0.1, 0.25\rangle}, \frac{x_3}{\langle 0.46, 0.3, 0.69\rangle}\right) \left((e_1, e_2), \frac{x_1}{\langle 0.32, 0.4, 0.59\rangle}, \frac{x_2}{\langle 0.65, 0.6, 0.4\rangle}, \frac{x_3}{\langle 0.30, 0.4, 0.47\rangle}\right) \left((e_3, e_1), \frac{x_1}{\langle 0.51, 0.9, 0.35\rangle}, \frac{x_2}{\langle 0.55, 0.7, 0.2\rangle}, \frac{x_3}{\langle 0.53, 0.2, 0.69\rangle}\right) \left((e_3, e_2), \frac{x_1}{\langle 0.44, 0.9, 0.49\rangle}, \frac{x_2}{\langle 0.65, 0.7, 0.4\rangle}, \frac{x_3}{\langle 0.3, 0.4, 0.47\rangle}\right)\right\}$$

- **Definition 23**

Consider two ENSSs $(\psi, \mathrm{E}_1)_{\Lambda_1}$and $(\phi, \mathrm{E}_2)_{\Lambda_2}$over the universe *U*. OR operation between these sets, written as $(\psi, \mathrm{E}_1)_{\Lambda_1} \vee (\phi, \mathrm{E}_2)_{\Lambda_2}$, is defined as

$$(\psi, \mathrm{E}_1)_{\Lambda_1} \vee (\phi, \mathrm{E}_2)_{\Lambda_2} = (\Sigma, \mathrm{E}_1 \times \mathrm{E}_2)_{\Lambda_s}$$

where $\Sigma_{\Lambda_s}(a, b) = (\psi(a) \cup \phi(b))_{\Lambda_s}$:$\forall(a, b) \in E_1 \times E_2$, $\cup$ is effective neutrosophic soft union and *s* represents any *s*-norm.

- **Example 7**

Consider example 6, we employ the basic fuzzy union operation to obtain Λs from Λ_1 and Λ_2 in the following manner:

$$\Lambda_s(x_1) = \left\{\frac{a_1}{0.7}, \frac{a_2}{0.5}, \frac{a_3}{1}, \frac{a_4}{1}\right\}, \Lambda_s(x_2) = \left\{\frac{a_1}{0.4}, \frac{a_2}{0.7}, \frac{a_3}{1}, \frac{a_4}{1}\right\},$$
$$\Lambda_s(x_3) = \left\{\frac{a_1}{0.7}, \frac{a_2}{0.5}, \frac{a_3}{0.6}, \frac{a_4}{0.9}\right\}$$

Through the utilization of the neutrosophic soft intersection, we obtain $(\psi, \mathrm{E}_1) \vee (\phi, \mathrm{E}_2) = (\Sigma, \mathrm{E}_1 \times \mathrm{E}_2)$ as follows:

$$(\Sigma, \mathrm{E}_1 \times \mathrm{E}_2) = \left\{\left((e_1, e_1), \frac{x_1}{\langle 0.5, 0.1, 0.4\rangle}, \frac{x_2}{\langle 0.3, 0, 0.4\rangle}, \frac{x_3}{\langle 0.4, 0.2, 0.5\rangle}\right), \left((e_1, e_2), \left\{\frac{x_1}{\langle 0.5, 0.2, 0.4\rangle}, \frac{x_2}{\langle 0.3, 0.1, 0.5\rangle}, \frac{x_3}{\langle 0.3, 0.3, 0.5\rangle}\right\}\right), \left((e_3, e_1), \frac{x_1}{\langle 0.6, 0.1, 0.3\rangle}, \frac{x_2}{\langle 0.4, 0, 0.1\rangle}, \frac{x_3}{\langle 0.8, 0.1, 0.2\rangle}\right), \left((e_3, e_2), \frac{x_1}{\langle 0.6,0.4, 0.3\rangle}, \frac{x_2}{\langle 0.4, 0.6, 0.1\rangle}, \frac{x_3}{\langle 0.8, 0.1, 0.2\rangle}\right)\right\}$$

Now, by using definitions 23 and 14 we get the ENSS $(\Sigma, \mathrm{E}_1 \times \mathrm{E}_2)_{\Lambda_s}$ as follows:

$$(\Sigma, \mathrm{E}_1 \times \mathrm{E}_2)_{\Lambda_s} = \left\{\left((e_1, e_1), \frac{x_1}{\langle 0.9, 0.1, 0.08\rangle}, \frac{x_2}{\langle 0.83, 0, 0.1\rangle}, \frac{x_3}{\langle 0.81, 0.2, 0.16\rangle}\right), \left((e_1, e_2), \frac{x_1}{\langle 0.9, 0.2, 0.08\rangle}, \frac{x_2}{\langle 0.84, 0.1, 0.11\rangle}, \frac{x_3}{\langle 0.77, 0.3, 0.16\rangle}\right)\right.$$

$$\left((e_3,e_1),\frac{x_1}{\langle 0.92,0.1,0.06\rangle},\frac{x_2}{\langle 0.87,0,0.02\rangle},\frac{x_3}{\langle 0.67,0.1,0.07\rangle}\right),$$
$$\left.\left((e_3,e_2),\frac{x_1}{\langle 0.92,0.4,0.06\rangle},\frac{x_2}{\langle 0.87,0.6,0.02\rangle},\frac{x_3}{\langle 0.94,0.1,0.07\rangle}\right)\right\}$$

6. An Application of ENSS in Decision Making Problem

In this section, we illustrate how ENSSs can be applied to handle decision-making challenges. To commence, we revisit some fundamental definitions.

- **Definition 24 [16]**

A comparison matrix is a matrix with rows labeled by object names h_1, h_2, …, h_n and columns labeled by parameters e_1, e_2, …, e_m. The entries c_{ij} are computed using the formula $c_{ij}=a+b-c$, where 'a' is the integer calculated as the count of how many times $T_{h_i}(e_j)$ exceeds or equal to $T_{h_k}(e_j)$', for $h_i \neq h_k$, $\forall h_k \in$U, 'b' is the integer calculated as the count of how many times $I_{h_i}(e_j)$ exceeds or equals to $I_{h_k}(e_j)$', is *for* $h_i \neq h_k, \forall h_k \in U$, and 'c' the integer calculated as the count of how many times $F_{h_i}(e_j)$ exceeds or equals $F_{h_k}(e_j)$' *for* $h_i \neq h_k, \forall h_k \in U$.

- **Definition 25[16]**

The score assigned to an object h_i, *for* all i is denoted as s_i and is computed as $s_i = \sum_j c_{ij}$. Subsequently, we introduce an algorithm designed for the optimal choice of an item.

6.1. Algorithm

As a combination of Alkhazaleh [4] algorithm for EFSS and Maji [16] algorithm for NSS, we get the following Algorithm (1) for ENSS:

Algorthim 1: ENSS

1. *Provide the effective sets of parameters Λ_1 and Λ_2 as input.*
2. *Input the NSS (H, A)*
3. *Provide the NSSs (ψ, E_1) and (ϕ, E_2) as input.*
4. *Obtain an effective set Λ_s from Λ_1 and Λ_2.*
5. *Obtain the NSS (σ, E) as required.*
6. *Calculate the resulting ENSS $(\sigma,E)_{\Lambda_s}$ accordingly.*
7. *Construct comparison table of ENSS.*
8. *Calculate the score s_i of h_i, for all i.*
9. *The optimal choice is $s_k = max_i s_i$*

6.2. Application in a Decision-Making Problem

- **Example 8**

Consider the set of mobile phones $U=\{x_1,x_2,x_3,x_4,x_5,x_6\}$ and the set of parameters $E=\{e_1, e_2, e_3, e_4, e_5, e_6\}$ where $E_1=\{e_1, e_3, e_4, e_6\}$ and $E_2=\{e_1, e_2, e_3, e_4, e_5\}$and e_1=Ram, e_2=Memory, e_3=Battery Power, e_4=Camera, e_5=Resolution and e_6=Water Proof. Consider the set of effective parameters $A=\{a_1, a_2, a_3, a_4\}$ such that a_1: Every component is produced in the primary factory a_2: It underwent reassembly at the primary factory, a_3: It was in the possession of a single owner, not shared by multiple individuals, and a_4: The software is operating in its most recent version. Consider the effective set over A for every x_i in U, as provided in the following manner:

$$\Lambda_1(x_1)=\left\{\frac{a_1}{0.8},\frac{a_2}{0.2},\frac{a_3}{0.1},\frac{a_4}{0.2}\right\},\ \Lambda_1(x_2)=\left\{\frac{a_1}{1},\frac{a_2}{0.2},\frac{a_3}{0.7},\frac{a_4}{0.2}\right\},$$
$$\Lambda_1(x_3)=\left\{\frac{a_1}{0.1},\frac{a_2}{0.2},\frac{a_3}{0.5},\frac{a_4}{0.1}\right\},\Lambda_1(x_4)=\left\{\frac{a_1}{0.2},\frac{a_2}{0.5},\frac{a_3}{0.3},\frac{a_4}{0.4}\right\},$$
$$\Lambda_1(x_5)=\left\{\frac{a_1}{0.2},\frac{a_2}{0.4},\frac{a_3}{0.4},\frac{a_4}{0.2}\right\},\Lambda_1(x_6)=\left\{\frac{a_1}{0.1},\frac{a_2}{0.1},\frac{a_3}{0.2},\frac{a_4}{0.5}\right\},$$
$$\Lambda_2(x_1)=\left\{\frac{a_1}{0.6},\frac{a_2}{1},\frac{a_3}{0.1},\frac{a_4}{0.3}\right\},\Lambda_2(x_2)=\left\{\frac{a_1}{0.5},\frac{a_2}{1},\frac{a_3}{0.3},\frac{a_4}{0.6}\right\},$$
$$\Lambda_2(x_3)=\left\{\frac{a_1}{0.3},\frac{a_2}{0.1},\frac{a_3}{0.8},\frac{a_4}{0.1}\right\},\Lambda_2(x_4)=\left\{\frac{a_1}{0.1},\frac{a_2}{0.8},\frac{a_3}{0.2},\frac{a_4}{0.9}\right\},$$
$$\Lambda_2(x_5)=\left\{\frac{a_1}{0.1},\frac{a_2}{0.5},\frac{a_3}{0.3},\frac{a_4}{0.3}\right\},\ \Lambda_2(x_6)=\left\{\frac{a_1}{0.2},\frac{a_2}{0.1},\frac{a_3}{0.3},\frac{a_4}{0.4}\right\}$$

Consider the NSSs given as follows:

$$(\psi,E_1)=$$
$$\left\{\left(e_1,\langle\frac{x_1}{\langle 0.5,0.3,0.8\rangle},\frac{x_2}{\langle 0.2,0.6,0.8\rangle},\frac{x_3}{\langle 0.1,0.1,0.5\rangle},\frac{x_4}{\langle 0.3,0.8,0.2\rangle},\frac{x_5}{\langle 0.6,0.5,0.3\rangle},\frac{x_6}{\langle 0.3,0.1,0.5\rangle}\rangle\right),\right.$$
$$\left(e_3,\langle\frac{x_1}{\langle 0.6,0.4,0.8\rangle},\frac{x_2}{\langle 0.4,0.3,0.6\rangle},\frac{x_3}{\langle 0.7,0.5,0.4\rangle},\frac{x_4}{\langle 0.8,0.6,0.7\rangle},\frac{x_5}{\langle 0.4,0.7,0.9\rangle},\frac{x_6}{\langle 0.1,0.1,0.9\rangle}\rangle\right),$$
$$\left(e_4,\langle\frac{x_1}{\langle 0.7,0.4,0.6\rangle},\frac{x_2}{\langle 0.4,0.3,0.8\rangle},\frac{x_3}{\langle 0.7,0.5,0.6\rangle},\frac{x_4}{\langle 0.4,0.8,0.6\rangle},\frac{x_5}{\langle 0.8,0.3,0.4\rangle},\frac{x_6}{\langle 0.4,0.2,0.7\rangle}\rangle\right),$$
$$\left.\left(e_6,\langle\frac{x_1}{\langle 0.5,0.6,0.8\rangle},\frac{x_2}{\langle 0.6,0.8,0.1\rangle},\frac{x_3}{\langle 0.5,0.2,0.6\rangle},\frac{x_4}{\langle 0.8,0.6,0.7\rangle},\frac{x_5}{\langle 0.7,0.2,0.6\rangle},\frac{x_6}{\langle 0.8,0.3,0.4\rangle}\rangle\right)\right\}$$

$$(\phi,E_2)=$$
$$\left\{\left(e_1,\langle\frac{x_1}{\langle 0.3,0.2,0.6\rangle},\frac{x_2}{\langle 0.3,0.4,0.5\rangle},\frac{x_3}{\langle 0.2,0.3,0.7\rangle},\frac{x_4}{\langle 0.4,0.6,0.7\rangle},\frac{x_5}{\langle 0.8,0.4,0.5\rangle},\frac{x_6}{\langle 0.5,0.2,0.3\rangle}\rangle\right),\right.$$
$$\left(e_2,\langle\frac{x_1}{\langle 0.7,0.2,0.6\rangle},\frac{x_2}{\langle 0.8,0.3,0.4\rangle},\frac{x_3}{\langle 0.7,0.2,0.2\rangle},\frac{x_4}{\langle 0.8,0.2,0.1\rangle},\frac{x_5}{\langle 0.7,0.8,0.3\rangle},\frac{x_6}{\langle 0.8,0.5,0.6\rangle}\rangle\right),$$
$$\left(e_3,\langle\frac{x_1}{\langle 0.4,0.3,0.9\rangle},\frac{x_2}{\langle 0.5,0.2,0.8\rangle},\frac{x_3}{\langle 0.6,0.3,0.5\rangle},\frac{x_4}{\langle 0.5,0.5,0.6\rangle},\frac{x_5}{\langle 0.6,0.8,0.2\rangle},\frac{x_6}{\langle 0.2,0.1,0.8\rangle}\rangle\right),$$
$$\left(e_4,\langle\frac{x_1}{\langle 0.5,0.2,0.8\rangle},\frac{x_2}{\langle 0.6,0.4,0.7\rangle},\frac{x_3}{\langle 0.4,0.5,0.7\rangle},\frac{x_4}{\langle 0.6,0.9,0.3\rangle},\frac{x_5}{\langle 0.6,0.1,0.5\rangle},\frac{x_6}{\langle 0.7,0.4,0.6\rangle}\rangle\right),$$
$$\left.\left(e_5,\langle\frac{x_1}{\langle 0.6,0.1,0.8\rangle},\frac{x_2}{\langle 0.8,0.5,0.2\rangle},\frac{x_3}{\langle 0.8,0.3,0.6\rangle},\frac{x_4}{\langle 0.7,0.3,0.4\rangle},\frac{x_5}{\langle 0.7,0.5,0.6\rangle},\frac{x_6}{\langle 0.7,0.6,0.8\rangle}\rangle\right)\right\}$$

We employ the basic fuzzy union operation to derive Λ_s from Λ_1 and Λ_2 in the following manner:

$$\Lambda_s(x_1)=\left\{\frac{a_1}{0.8},\frac{a_2}{1},\frac{a_3}{0.1},\frac{a_4}{0.3}\right\},\Lambda_s(x_2)=\left\{\frac{a_1}{1},\frac{a_2}{1},\frac{a_3}{0.7},\frac{a_4}{0.6}\right\},$$
$$\Lambda_s(x_3)=\left\{\frac{a_1}{0.3},\frac{a_2}{0.2},\frac{a_3}{0.8},\frac{a_4}{0.1}\right\},\Lambda_s(x_4)=\left\{\frac{a_1}{0.2},\frac{a_2}{0.8},\frac{a_3}{0.3},\frac{a_4}{0.9}\right\},$$
$$\Lambda_s(x_5)=\left\{\frac{a_1}{0.2},\frac{a_2}{0.5},\frac{a_3}{0.4},\frac{a_4}{0.3}\right\},\ \Lambda_s(x_6)=\left\{\frac{a_1}{0.2},\frac{a_2}{0.1},\frac{a_3}{0.3},\frac{a_4}{0.5}\right\}$$

Then, we determine (σ, E)=(ψ, E_1)∪(ϕ, E_2) in the following manner:

$$(\sigma,E)=\left\{\left(e_1,\frac{x_1}{\langle 0.5,0.2,0.6\rangle},\frac{x_2}{\langle 0.3,0.4,0.5\rangle},\frac{x_3}{\langle 0.2,0.1,0.5\rangle},\frac{x_4}{\langle 0.4,0.6,0.2\rangle},\frac{x_5}{\langle 0.8,0.4,0.3\rangle},\frac{x_6}{\langle 0.5,0.1,0.3\rangle}\right),\right.$$
$$\left(e_2,\frac{x_1}{\langle 0.7,0.2,0.6\rangle},\frac{x_2}{\langle 0.8,0.3,0.4\rangle},\frac{x_3}{\langle 0.7,0.2,0.2\rangle},\frac{x_4}{\langle 0.8,0.2,0.1\rangle},\frac{x_5}{\langle 0.7,0.8,0.3\rangle},\frac{x_6}{\langle 0.8,0.5,0.6\rangle}\right),$$
$$\left(e_3,\frac{x_1}{\langle 0.6,0.3,0.8\rangle},\frac{x_2}{\langle 0.5,0.2,0.6\rangle},\frac{x_3}{\langle 0.7,0.3,0.4\rangle},\frac{x_4}{\langle 0.8,0.5,0.6\rangle},\frac{x_5}{\langle 0.6,0.7,0.2\rangle},\frac{x_6}{\langle 0.2,0.1,0.8\rangle}\right),$$
$$\left(e_4,\frac{x_1}{\langle 0.7,0.2,0.6\rangle},\frac{x_2}{\langle 0.6,0.3,0.7\rangle},\frac{x_3}{\langle 0.7,0.5,0.6\rangle},\frac{x_4}{\langle 0.6,0.8,0.3\rangle},\frac{x_5}{\langle 0.8,0.1,0.4\rangle},\frac{x_6}{\langle 0.7,0.2,0.6\rangle}\right),$$
$$\left(e_5,\frac{x_1}{\langle 0.6,0.1,0.8\rangle},\frac{x_2}{\langle 0.8,0.5,0.2\rangle},\frac{x_3}{\langle 0.8,0.3,0.6\rangle},\frac{x_4}{\langle 0.7,0.3,0.4\rangle},\frac{x_5}{\langle 0.7,0.5,0.6\rangle},\frac{x_6}{\langle 0.7,0.6,0.8\rangle}\right)$$
$$\left.\left(e_6,\frac{x_1}{\langle 0.5,0.6,0.8\rangle},\frac{x_2}{\langle 0.6,0.8,0.1\rangle},\frac{x_3}{\langle 0.5,0.2,0.6\rangle},\frac{x_4}{\langle 0.8,0.6,0.7\rangle},\frac{x_5}{\langle 0.7,0.2,0.6\rangle},\frac{x_6}{\langle 0.8,0.3,0.4\rangle}\right)\right\}$$

Then the ENSS is given as follows:

$$(\sigma,\mathrm{E})_{\Lambda_s}=\Big\{\Big(e_1,\frac{x_1}{\langle 0.78,0.2,0.27\rangle},\frac{x_2}{\langle 0.88,0.4,0.09\rangle},\frac{x_3}{\langle 0.48,0.1,0.32\rangle},\frac{x_4}{\langle 0.73,0.6,0.09\rangle},\frac{x_5}{\langle 0.87,0.4,0.2\rangle},\frac{x_6}{\langle 0.64,0.1,0.22\rangle}\Big),$$
$$\Big(e_2,\frac{x_1}{\langle 0.87,0.2,0.27\rangle},\frac{x_2}{\langle 0.97,0.3,0.07\rangle},\frac{x_3}{\langle 0.81,0.2,0.13\rangle},\frac{x_4}{\langle 0.91,0.2,0.05\rangle},\frac{x_5}{\langle 0.81,0.8,0.2\rangle},\frac{x_6}{\langle 0.86,0.5,0.44\rangle}\Big),$$
$$\Big(e_3,\frac{x_1}{\langle 0.82,0.3,0.36\rangle},\frac{x_2}{\langle 0.91,0.2,0.11\rangle},\frac{x_3}{\langle 0.81,0.3,0.26\rangle},\frac{x_4}{\langle 0.91,0.5,0.27\rangle},\frac{x_5}{\langle 0.74,0.7,0.13\rangle},\frac{x_6}{\langle 0.42,0.1,0.58\rangle}\Big),$$
$$\Big(e_4,\frac{x_1}{\langle 0.87,0.2,0.27\rangle},\frac{x_2}{\langle 0.93,0.3,0.12\rangle},\frac{x_3}{\langle 0.81,0.5,0.39\rangle},\frac{x_4}{\langle 0.82,0.8,0.14\rangle},\frac{x_5}{\langle 0.87,0.1,0.26\rangle},\frac{x_6}{\langle 0.78,0.2,0.44\rangle}\Big),$$
$$\Big(e_5,\frac{x_1}{\langle 0.82,0.1,0.36\rangle},\frac{x_2}{\langle 0.97,0.5,0.04\rangle},\frac{x_3}{\langle 0.87,0.3,0.39\rangle},\frac{x_4}{\langle 0.87,0.3,0.18\rangle},\frac{x_5}{\langle 0.81,0.5,0.39\rangle},\frac{x_6}{\langle 0.78,0.6,0.58\rangle}\Big),$$
$$\Big(e_6,\frac{x_1}{\langle 0.78,0.6,0.36\rangle},\frac{x_2}{\langle 0.93,0.8,0.02\rangle},\frac{x_3}{\langle 0.68,0.2,0.39\rangle},\frac{x_4}{\langle 0.91,0.6,0.32\rangle},\frac{x_5}{\langle 0.81,0.2,0.39\rangle},\frac{x_6}{\langle 0.86,0.3,0.29\rangle}\Big)\Big\}$$

The ENSS (σ, E). is presented in Table 1.

Table 1. Shows the tabular representation of (σ, E).

U \ E	e_1	e_2	e_3	e_4	e_5	e_6
x_1	⟨0.5,0.2,0.6⟩	⟨0.7,0.2,0.6⟩	⟨0.6,0.3,0.8⟩	⟨0.7,0.2,0.6⟩	⟨0.6,0.1,0.8⟩	⟨0.5,0.6,0.8⟩
x_2	⟨0.3,0.4,0.5⟩	⟨0.8,0.3,0.4⟩	⟨0.5,0.2,0.6⟩	⟨0.6,0.3,0.7⟩	⟨0.8,0.5,0.2⟩	⟨0.6,0.8,0.1⟩
x_3	⟨0.2,0.1,0.5⟩	⟨0.7,0.2,0.2⟩	⟨0.7,0.3,0.4⟩	⟨0.7,0.5,0.6⟩	⟨0.8,0.3,0.6⟩	⟨0.5,0.2,0.6⟩
x_4	⟨0.4,0.6,0.2⟩	⟨0.8,0.2,0.1⟩	⟨0.8,0.5,0.6⟩	⟨0.6,0.8,0.3⟩	⟨0.7,0.3,0.4⟩	⟨0.8,0.6,0.7⟩
x_5	⟨0.8,0.4,0.3⟩	⟨0.7,0.8,0.3⟩	⟨0.6,0.7,0.2⟩	⟨0.8,0.1,0.4⟩	⟨0.7,0.5,0.6⟩	⟨0.7,0.2,0.6⟩
x_6	⟨0.5,0.1,0.3⟩	⟨0.8,0.5,0.6⟩	⟨0.2,0.1,0.8⟩	⟨0.7,0.2,0.6⟩	⟨0.7,0.6,0.8⟩	⟨0.8,0.3,0.4⟩

Through the utilization of Maji algorithm [16], we determine the comparison table and score table for (σ, E) as presented in Tables 2 and 3 respectively002E

Table 2. Comparison table of (σ, E).

U \ E	e_1	e_2	e_3	e_4	e_5	e_6
x_1	1	-1	1	2	-5	0
x_2	1	5	-1	-1	9	7
x_3	-3	4	6	4	4	-1
x_4	7	7	6	6	4	4
x_5	7	5	8	4	4	1
x_6	3	4	-5	3	3	6

Table 3. Score table s_i of (σ, E).

U	S_i
x_1	-2
x_2	20
x_3	14
x_4	34
x_5	29
x_6	14

The optimal selection is x_4.Thus, based on the Maji algorithm [16], we determine that phone 4 is the optimal choice for NSS.

The ENSS $(\sigma, E)_{\Lambda_s}$. is presented in Tables 4.

Table 4. Tabular representation of $(\sigma, E)_{\Lambda_s}$.

U \ E	e_1	e_2	e_3	e_4	e_5	e_6
x_1	<0.78,0.2,0.27>	⟨0.87,0.2,0.27⟩	⟨0.82,0.3,0.36⟩	⟨0.87,0.2,0.27⟩	⟨0.82,0.1,0.36⟩	⟨0.78,0.6,0.36⟩
x_2	⟨0.88,0.4,0.09⟩	⟨0.97,0.3,0.07⟩	⟨0.91,0.2,0.11⟩	⟨0.93,0.3,0.12⟩	⟨0.97,0.5,0.04⟩	⟨0.93,0.8,0.02⟩
x_3	⟨0.48,0.1,0.32⟩	⟨0.81,0.2,0.13⟩	⟨0.81,0.3,0.26⟩	⟨0.81,0.5,0.39⟩	⟨0.87,0.3,0.39⟩	⟨0.68,0.2,0.39⟩
x_4	⟨0.73,0.6,0.09⟩	⟨0.91,0.2,0.05⟩	⟨0.91,0.5,0.27⟩	⟨0.82,0.8,0.14⟩	⟨0.87,0.3,0.18⟩	⟨0.91,0.6,0.32⟩
x_5	⟨0.87,0.4,0.2⟩	⟨0.81,0.8,0.2⟩	⟨0.74,0.7,0.13⟩	⟨0.87,0.1,0.26⟩	⟨0.81,0.5,0.39⟩	⟨0.81,0.2,0.39⟩
x_6	⟨0.64,0.1,0.22⟩	⟨0.86,0.5,0.44⟩	⟨0.42,0.1,0.58⟩	⟨0.78,0.2,0.44⟩	⟨0.78,0.6,0.58⟩	⟨0.86,0.3,0.29⟩

By using Algorithm (1), we find the comparison table and score table of $(\sigma, E)_{\Lambda_s}$ as in Tables 5 and 6 respectively.

Table 5. Comparison table of $(\sigma, E)_{\Lambda_s}$.

U \ E	e_1	e_2	e_3	e_4	e_5	e_6
x_1	1	1	2	3	0	2
x_2	8	7	6	8	9	10
x_3	-4	1	3	1	2	-4
x_4	6	6	6	6	5	6
x_5	6	3	5	2	1	-2
x_6	-1	2	-5	-3	0	4

Table 6. Score table S_i of $(\sigma, E)_{\Lambda_s}$

U	S_i
x_1	9
x_2	48
x_3	-1
x_4	35
x_5	18
x_6	-3

Our decision to choose mobile phone 2 is evident, given that it achieves the highest score of 48, attributed to x_2. However, upon comparison with the Maji algorithm [16] employed for NSS without effective parameters, we deduce that the ENSS has altered the decision from phone 4 to phone 2.

7. An Application of ENSS in Medical Diagnosis

- **Example 9**

Suppose that a group of four patients, denoted as $P=\{p_1, p_2, p_3, p_4\}$, who are currently hospitalized due to illness. The hospital's diagnostic specialist has listed specific symptoms to diagnose the patients' conditions. $S=\{s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9, s_{10}, s_{11}, s_{12}, s_{13}, s_{14}, s_{15}, s_{16}, s_{17}, s_{18}, s_{19}, s_{20}\}$. Here, s_1= Fever, s_2=dry cough, s_3=loose motion, s_4=breathing difficulty or experiencing shortness of breath, s_5=headache, s_6=tiredness,

s_7=aches, s_8=runny nose, s_9=sore throat, s_{10}=acute pneumonia, s_{11}=rash, s_{12}=diarrhoea, s_{13}=pain in the bones and joints, s_{14}=nausea, s_{15}=vomiting, s_{16}=pain behind the eyes, s_{17}=chills, s_{18}=sweating, s_{19}=abdominal pain and s_{20}=swollen glands.

Furthermore, let D={d_1, d_2, d_3, d_4} represent a group of illnesses, where COVID-19, d_2=dengue fever, d_3=malaria, d_4=typhoid. Consider A={a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9} where a_1=During the two weeks prior, he journeyed to a nation in Europe, America, Africa, or East Asia, a_2=He came into close proximity (within 6 feet) with an individual who has COVID-19, a_3=He works at medical institutions, a_4=He regularly utilizes public transportation as part of his job routine, a_5=He eats out at restaurants or indulges in fast food, a_6=He was situated in an area with calm water, especially during the early morning and evening hours, a_7=He used to sleep without any covering or protection from mosquitoes, a_8=Eating food that is raw or inadequately cooked, and a_9=Eating foods and beverages purchased from roadside vendors.

We identify the everyday activities and lifestyles of the sick individuals as delineated in Table 7.

Table 7. The everyday activities and lifestyles of the sick individuals.

A \ p_i	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
p_1	Yes	Yes	No	Yes	Yes	No	No	No	Yes
p_2	No	No	Yes	No	Yes	No	Yes	Yes	Yes
p_3	Yes	No	No	No	No	Yes	Yes	No	No
p_4	No	No	No	Yes	Yes	No	No	Yes	No

The association between the mentioned parameters and the specified disease is depicted in Table 8 in the following manner:

Table 8. The relationship between parameters and diseases.

A \ D	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	$\lvert A_j \rvert$
d_1	Yes	Yes	Yes	Yes	No	No	No	No	No	4
d_2	Yes	No	No	No	No	Yes	Yes	No	No	3
d_3	Yes	No	No	No	No	Yes	Yes	No	No	3
d_4	Yes	No	No	No	Yes	No	No	Yes	Yes	4

Tables ranging from 9 to 12 depict $\Lambda_{d_j}(p_i)$ for each patient concerning the designated diseases, as described below:

Table 9. Table presentation of $\Lambda_{d_1}(p_i)$.

A \ p_i	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	Sum
$p_1 d_1$	1	1	0	1	0	0	0	0	0	3
$p_2 d_1$	0	0	1	0	0	0	0	0	0	1
$p_3 d_1$	1	0	0	0	0	0	0	0	0	1
$p_4 d_1$	0	0	0	0	1	0	0	0	0	1

Table 10. Table presentation of $\Lambda_{d_2}(p_i)$.

A \ p_i	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	Sum
$p_1 d_2$	1	0	0	0	0	0	0	0	0	1
$p_2 d_2$	0	0	0	0	0	0	1	0	0	1
$p_3 d_2$	1	0	0	0	0	1	1	0	0	3
$p_4 d_2$	0	0	0	0	0	0	0	0	0	0

Table 11. Table presentation of $\Lambda_{d_3}(p_i)$.

A \ p_i	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	Sum
$p_1 d_3$	1	0	0	0	0	0	0	0	0	1
$p_2 d_3$	0	0	0	0	0	0	0	1	0	1
$p_3 d_3$	1	0	0	0	0	1	1	0	0	3
$p_4 d_3$	0	0	0	0	0	0	0	0	0	0

Table 12. Table presentation of $\Lambda_{d_4}(p_i)$.

A \ p_i	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	Sum
$p_1 d_4$	1	0	0	0	1	0	0	0	1	3
$p_2 d_4$	0	0	0	0	1	0	0	1	0	2
$p_3 d_4$	1	0	0	0	0	0	0	0	1	2
$p_4 d_4$	0	0	0	0	1	0	0	1	1	3

Suppose the tabular representation of (ψ, S) patient symptom given in Tables 13 to 16.

Table 13. Table presentation of (ψ, S) 1st part.

S \ p	s_1	s_2	s_3	s_4	s_5
p_1	<0.7,0.2,0.5>	<0.3,0.1,0.2>	<0.3,0.1,0.5>	<0.2,0.3,0.5>	<0.8,0.3,0.6>
p_2	<0.4,0.1,0.2>	<0.6,0,0.3>	<0.7,0.2,0.5>	<0.5,0.1,0.2>	<0.3,0,0.2>
p_3	<0.9,0.1,0.4>	<0.5,0.1,0.3>	<0.5,0.2,0.7>	<0.3,0.4,0.5>	<0.8,0.2,0.6>
p_4	<0.6,0.1,0.3>	<0.8,0.1,0.2>	<0.8,0.4,0.7>	<0.7,0.2,0.3>	<0.5,0.3,0.4>

Table 14. Table presentation of (ψ, S) 2nd part.

S \ p	s_6	s_7	s_8	s_9	s_{10}
P_1	<0.9,0.1,0.5>	<0.3,0,0.8>	<0.8,0.2,0.6>	<0.7,0.1,0.5>	<0.5,0.1,0.8>
P_2	<0.8,0.4,0.2>	<0.3,0.1,0.7>	<0.6,0.4,0.8>	<0.3,0.5,0.1>	<0.2,0.3,0.7>
P_3	<0.9,0.2,0.5>	<0.4,0.1,0.8>	<0.5,0.4,0.6>	<0.5,0.2,0.7>	<0.4,0.3,0.6>
P_4	<0.6,0.4,0.7>	<0.3,0.4,0.7>	<0.6,0.5,0.4>	<0.5,0.4,0.8>	<0.8,0.4,0.5>

Table 15. Table presentation of (ψ, S) 3rd part.

S \ p	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}
P_1	<0.6,0.2,0.3>	<0.8,0.3,0.4>	<0.7,0.1,0.4>	<0.3,0.2,0.9>	<0.1,0.3,0.5>
P_2	<0.9,0.2,0.6>	<0.4,0.2,0.5>	<0.3,0.1,0.7>	<0.2,0,0.3>	<0.7,0.2,0.4>
P_3	<0.8,0.1,0.5>	<0.7,0.2,0.4,>	<0.9,0.2,0.1>	<0.6,0.1,0.4>	<0.1,0.2,0.7>
P_4	<0.9,0.3,0.2>	<0.5,0.3,0.6>	<0.5,0.1,0.4>	<0.7,0.3,0.5>	<0.7,0.3,0.4>

Table 16. Table presentation of (ψ, S) 4th part.

S \ p	s_{16}	s_{17}	s_{18}	s_{19}	s_{20}
P_1	<0.2,0.1,0.4>	<0.8,0.4,0.6>	<0.5,0.3,0.7>	<0.4,0,0.2>	<0.1,0.6,0.3>
P_2	<0.2,0.3,0.9>	<0.1,0.5,0.3>	<0.2,0.2,0.9>	<0.4,0.2,0.7>	<0.8,0.4,0.5>
P_3	<0.3,0.2,0.5>	<0.5,0.3,0.7,>	<0.4,0.2,0.8>	<0.7,0.5,0.6>	<0.3,0.5,0.4>
P_4	<0.7,0.3,0.9>	<0.2,0.5,0.3>	<0.1,0.1,0.7>	<0.5,0.2,0.4>	<0.8,0.4,0.6>

The table presentation of (ϕ, S) representing the model symptoms, is provided in the tables from Tables 17 to 20.

Table 17. Table presentation of *(ϕ, S)* 1st part.

S \ D	s_1	s_2	s_3	s_4	s_5
d_1	<1,1,0>	<1,1,0>	<1,1,0>	<1,1,0>	<0.5,0.5,0.5>
d_2	<0,0,1>	<0,0,1>	<0,0,1>	<0.5,0.5,0.5>	<1,1,0>
d_3	<1,1,0>	<0.5,0.5,0.5,>	<0,0,1>	<0,0,1>	<1,1,0>
d_4	<1,1,0>	<0.5,0.5,0.5>	<0,0,1>	<0,0,1>	<,1,1,0>

Table 18. Table presentation of *(ϕ, S)* 2nd part.

S \ D	s_6	s_7	s_8	s_9	s_{10}
d_1	<0,0,1>	<0.5,0.5,0.5>	<0.5,0.5,0.5>	<0.5,0.5,0.5>	<1,1,0>
d_2	<0.5,0.5,0.5>	<0,0,1>	<0,0,1>	<0,0,1>	<0,0,1>
d_3	<0,0,1>	<1,1,0>	<0,0,1>	<0,0,1>	<0,0,1>
d_4	<1,1,0>	<1,1,0>	<0,0,1>	<0,0,1>	<0,0,1>

Table 19. Table presentation of (ϕ, S) 3[rd] part.

D \ S	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}
d_1	<0,0,1>	<0.5,0.5,0.5>	<0,0,1>	<0,0,1>	<0,0,1>
d_2	<1,1,0>	<0,0,1>	<1,1,0>	<1,1,0>	<1,1,0>
d_3	<0,0,1>	<0,0,1>	<1,1,0>	<1,1,0>	<1,1,0>
d_4	<0.5,0.5,0.5>	<1,1,0>	<1,1,0>	<1,1,0>	<0,0,1>

Table 20. Table presentation of (ϕ, S) 4[th] part.

D \ S	s_{16}	s_{17}	s_{18}	s_{19}	s_{20}
d_1	<0,0,1>	<0,0,1>	<0,0,1>	<0,0,1>	<0,0,1>
d_2	<1,1,0>	<0,0,1>	<0,0,1>	<0.5,0.5,0.5>	<1,1,0>
d_3	<0,0,1>	<1,1,0>	<0.5,0.5,0.5>	<0.5,0.5,0.5>	<0,0,1>
d_4	<0,0,1>	<0.5,0.5,0.5>	<0.5,0.5,0.5>	<1,1,0>	<0,0,1>

We form the ENSS utilizing definition 14 and the information from Tables 13 to 16, presented in Tables 21 to 40.

Table 21. Table presentation of $(\psi, S)_{\Lambda_{d_1}}$ 1[st] part.

P \ S	s_1	s_2	s_3	s_4
p_1	<0.93,0.2,0.13>	<0.83,0.1,0.05>	<0.83,0.1,0.13>	<0.83,0.1,0.13>
p_2	<0.55,0.1,0.15>	<0.7,0,0.23>	<0.78,0.2,0.38>	<0.63,0.1,0.15>
p_3	<0.93,0.1,0.3>	<0.63,0.1,0.23>	<0.63,0.2,0.53>	<0.48,0.4,0.38>
p_4	<0.7,0.1,0.23>	<0.85,0.1,0.15>	<0.85,0.4,0.45>	<0.78,0.2,0.23>

Table 22. Table presentation of $(\psi, S)_{\Lambda_{d_1}}$ 2[nd] part.

P \ S	s_5	s_6	s_7	s_8
p_1	<0.95,0.3,0.15>	<0.98,0.1,0.13>	<0.83,0,0.2>	<0.95,0.2,0.15>
p_2	<0.48,0,0.15>	<0.85,0.4,0.15>	<0.48,0.1,0.53>	<0.7,0.4,0.6>
p_3	<0.85,0.2,0.45>	<0.93,0.2,0.38>	<0.55,0.1,0.6>	<0.63,0.4,0.45>
p_4	<0.63,0.3,0.3>	<0.7,0.4,0.53>	<0.48,0.4,0.53>	<0.7,0.5,0.3>

Table 23. Table presentation of $(\psi, S)_{\Lambda_{d_1}}$ 3[rd] part.

P \ S	s_9	s_{10}	s_{11}	s_{12}
p_1	<0.93,0.1,0.13>	<0.88,0.1,0.2>	<0.9,0.2,0.08>	<0.95,0.3,0.1>
p_2	<0.48,0.5,0.08>	<0.4,0.3,0.53>	<0.93,0.2.0.45>	<0.55,0.2,0.38>
p_3	<0.63,0.2,0.53>	<0.55,0.3,0.45>	<0.85,0.1,0.38>	<0.78,0.2,0.3>
p_4	<0.53,0.4,0.6>	<0.85,0.4,0.38>	<0.93,0.3,0.15>	<0.63,0.3,0.45>

Table 24. Table presentation of $(\psi, S)_{\Lambda_{d_1}}$ 4[th] part.

P \ S	s_{13}	s_{14}	s_{15}	s_{16}
p_1	<0.93,0.1,0.1>	<0.83,0.2,0.23>	<0.78,0.3,0.13>	<0.8,0.1,0.1>
p_2	<0.48,0.1,0.53>	<0.4,0,0.23>	<0.78,0.2.0.3>	<0.4,0.3,0.68>
p_3	<0.93,0.2,0.08>	<0.7,0.1,0.3>	<0.33,0.2,0.53>	<0.48,0.2,0.38>
p_4	<0.63,0.1,0.3>	<0.78,0.3,0.38>	<0.78,0.3,0.3>	<0.78,0.3,0.68>

Table 25. Table presentation of $(\psi, S)_{\Lambda_{d_1}}$ 5[th] part.

P \ S	s_{17}	s_{18}	s_{19}	s_{20}
p_1	<0.95,0.4,0.15>	<0.88,0.3,0.18>	<0.85,0,0.05>	<0.78,0.6,0.08>
p_2	<0.33,0.5,0.23>	<0.4,0.2,0.68>	<0.55,0.2.0.53>	<0.85,0.4,0.38>
p_3	<0.63,0.3,0.53>	<0.55,0.2,0.6>	<0.78,0.5,0.45>	<0.48,0.5,0.3>
p_4	<0.4,0.5,0.23>	<0.33,0.1,0.53>	<0.63,0.2,0.3>	<0.85,0.4,0.45>

Table 26. Table presentation of $(\psi, S)_{\Lambda_{d_2}}$ 1[st] part.

P \ S	s_1	s_2	s_3	s_4
p_1	<0.8,0.2,0.33>	<0.53,0.1,0.13>	<0.53,0.1,0.33>	<0.47,0.3,0.33>
p_2	<0.6,0.1,0.13>	<0.73,0,0.2>	<0.8,0.2,0.33>	<0.67,0.1,0.13>
p_3	<1,0.1,0>	<1,0.1,0>	<1,0.2,0>	<1,0.4,0>
p_4	<0.6,0.1,0.3>	<0.8,0.1,0.2>	<0.8,0.4,0.6>	<0.7,0.2,0.3>

Table 27. Table presentation of $(\psi, S)_{\Lambda_{d_2}}$ 2[nd] part.

P \ S	s_5	s_6	s_7	s_8
p_1	<0.87,0.3,0.4>	<0.93,0.1,0.33>	<0.53,0,0.53>	<0.87,0.2,0.4>
p_2	<0.53,0,0.13>	<0.87,0.4,0.13>	<0.53,0.1,0.47>	<0.73,0.4,0.53>
p_3	<1,0.2,0>	<1,0.2,0>	<1,0.1,0>	<1,0.4,0>
p_4	<0.5,0.3,0.4>	<0.6,0.4,0.7>	<0.3,0.4,0.7>	<0.6,0.5,0.4>

Table 28. Table presentation of $(\psi, S)_{\Lambda_{d_2}}$ 3[rd] part.

P \ S	s_9	s_{10}	s_{11}	s_{12}
p_1	<0.8,0.1,0.33>	<0.67,0.1,0.53>	<0.73,0.2,0.2>	<0.87,0.3,0.27>
p_2	<0.53,0.5,0.07>	<0.47,0.3,0.47>	<0.93,0.2,0.4>	<0.6,0.2,0.33>
p_3	<1,0.2,0>	<1,0.3,0>	<1,0.1,0>	<1,0.2,0>
p_4	<0.5,0.4,0.8>	<0.8,0.4,0.5>	<0.9,0.3,0.2>	<0.5,0.3,0.6>

Table 29. Table presentation of $(\psi, S)_{\Lambda_{d_2}}$ 4[th] part.

P \ S	s_{13}	s_{14}	s_{15}	s_{16}
p_1	<0.8,0.1,0.27>	<0.53,0.2,0.6>	<0.4,0.3,0.33>	<0.47,0.1,0.27>
p_2	<0.53,0.1,0.47>	<0.47,0,0.2>	<0.8,0.2,0.27>	<0.47,0.3,0.6>
p_3	<1,0.2,0>	<1,0.1,0>	<1,0.2,0>	<1,0.2,0>
p_4	<0.5,0.1,0.4>	<0.7,0.3,0.5>	<0.7,0.3,0.4>	<0.7,0.3,0.9>

Table 30. Table presentation of $(\psi, S)_{\Lambda_{d_2}}$ 5[th] part.

P \ S	s_{17}	s_{18}	s_{19}	s_{20}
p_1	<0.87,0.4,0.4>	<0.67,0.3,0.47>	<0.6,0,0.13>	<0.4,0.6,0.2>
p_2	<0.4,0.5,0.2>	<0.47,0.2,0.6>	<0.6,0.2,0.47>	<0.87,0.4,0.33>
p_3	<1,0.3,0>	<1,0.2,0>	<1,0.5,0>	<1,0.5,0>
p_4	<0.2,0.5,0.3>	<0.1,0.1,0.7>	<0.5,0.2,0.4>	<0.8,0.4,0.6>

Table 31. Table presentation of $(\psi, S)_{\Lambda_{d_3}}$ 1[st] part.

P \ S	s_1	s_2	s_3	s_4
p_1	<0.8,0.2,0.33>	<0.53,0.1,0.13>	<0.53,0.1,0.33>	<0.47,0.3,0.33>
p_2	<0.6,0.1,0.13>	<0.73,0,0.2>	<0.8,0.2,0.33>	<0.67,0.1,0.13>
p_3	<1,0.1,0>	<1,0.1,0>	<1,0.2,0>	<1,0.4,0>
p_4	<0.6,0.1,0.3>	<0.8,0.1,0.2>	<0.8,0.4,0.6>	<0.7,0.2,0.3>

Table 32. Table presentation of $(\psi, S)_{\Lambda_{d_3}}$ 2[nd] part.

P \ S	s_5	s_6	s_7	s_8
p_1	<0.87,0.3,0.4>	<0.93,0.1,0.33>	<0.53,0,0.53>	<0.87,0.2,0.4>
p_2	<0.53,0,0.13>	<0.87,0.4,0.13>	<0.53,0.1,0.47>	<0.73,0.4,0.53>
p_3	<1,0.2,0>	<1,0.2,0>	<1,0.1,0>	<1,0.4,0>
p_4	<0.5,0.3,0.4>	<0.6,0.4,0.7>	<0.3,0.4,0.7>	<0.6,0.5,0.4>

Table 33. Table presentation of $(\psi, S)_{\Lambda_{d_3}}$ 3[rd] part.

P \ S	s_9	s_{10}	s_{11}	s_{12}
p_1	<0.8,0.1,0.33>	<0.67,0.1,0.53>	<0.73,0.2,0.2>	<0.87,0.3,0.27>
p_2	<0.53,0.5,0.07>	<0.47,0.3,0.47>	<0.93,0.2,0.4>	<0.6,0.2,0.33>
p_3	<1,0.2,0>	<1,0.3,0>	<1,0.1,0>	<1,0.2,0>
p_4	<0.5,0.4,0.8>	<0.8,0.4,0.5>	<0.9,0.3,0.2>	<0.5,0.3,0.6>

Table 34. Table presentation of $(\psi, S)_{\Lambda_{d_3}}$ 4[th] part.

P \ S	s_{13}	s_{14}	s_{15}	s_{16}
p_1	<0.8,0.1,0.27>	<0.53,0.2,0.6>	<0.4,0.3,0.33>	<0.47,0.1,0.27>
p_2	<0.53,0.1,0.47>	<0.47,0,0.2>	<0.8,0.2,0.27>	<0.47,0.3,0.6>
p_3	<1,0.2,0>	<1,0.1,0>	<1,0.2,0>	<1,0.2,0>
p_4	<0.5,0.1,0.4>	<0.7,0.3,0.5>	<0.7,0.3,0.4>	<0.7,0.3,0.9>

Table 35. Table presentation of $(\psi, S)_{\Lambda_{d_3}}$ 5[th] part.

P \ S	s_{17}	s_{18}	s_{19}	s_{20}
p_1	<0.87,0.4,0.4>	<0.67,0.3,0.47>	<0.6,0,0.13>	<0.4,0.6,0.2>
p_2	<0.4,0.5,0.2>	<0.47,0.2,0.6>	<0.6,0.2,0.47>	<0.87,0.4,0.33>
p_3	<1,0.3,0>	<1,0.2,0>	<1,0.5,0>	<1,0.5,0>
p_4	<0.2,0.5,0.3>	<0.1,0.1,0.7>	<0.5,0.2,0.4>	<0.8,0.4,0.6>

Table 36. Table presentation of $(\psi,S)_{\Lambda_{d_4}}$ 1st part.

S / P	s_1	s_2	s_3	s_4
p_1	<0.93,0.2,0.13>	<0.83,0.1,0.05>	<0.83,0.1,0.13>	<0.8,0.3,0.13>
p_2	<0.85,0.1,0.05>	<0.9,0,0.08>	<0.93,0.2,0.13>	<0.88,0.1,0.05>
p_3	<0.93,0.1,0.3>	<0.63,0.1,0.23>	<0.63,0.2,0.53>	<0.48,0.4,0.38>
p_4	<0.8,0.1,0.15>	<0.9,0.1,0.1>	<0.9,0.4,0.3>	<0.85,0.2,0.15>

Table 37. Table presentation of $(\psi,S)_{\Lambda_{d_4}}$ 2nd part.

S / P	s_5	s_6	s_7	s_8
p_1	<0.95,0.3,0.15>	<0.98,0.1,0.13>	<0.83,0,0.2>	<0.95,0.2,0.15>
p_2	<0.83,0,0.05>	<0.95,0.4,0.05>	<0.83,0.1,0.18>	<0.9,0.4,0.2>
p_3	<0.85,0.2,0.45>	<0.93,0.2,0.38>	<0.55,0.1,0.6>	<0.63,0.4,0.45>
p_4	<0.75,0.3,0.22>	<0.8,0.4,0.35>	<0.65,0.4,0.35>	<0.8,0.5,0.2>

Table 38. Table presentation of $(\psi,S)_{\Lambda_{d_4}}$ 3rd part.

S / P	s_9	s_{10}	s_{11}	s_{12}
p_1	<0.93,0.1,0.13>	<0.88,0.1,0.2>	<0.9,0.2,0.08>	<0.95,0.3,0.1>
p_2	<0.83,0.5,0.03>	<0.8,0.3,0.18>	<0.98,0.2,0.15>	<0.85,0.2,0.13>
p_3	<0.63,0.2,0.53>	<0.55,0.3,0.45>	<0.85,0.1,0.38>	<0.78,0.2,0.3>
p_4	<0.75,0.4,0.4>	<0.9,0.4,0.25>	<0.95,0.3,0.1>	<0.75,0.3,0.3>

Table 39. Table presentation of $(\psi,S)_{\Lambda_{d_4}}$ 4th part.

S / P	s_{13}	s_{14}	s_{15}	s_{16}
p_1	<0.93,0.1,0.1>	<0.83,0.2,0.23>	<0.78,0.3,0.13>	<0.8,0.1,0.1>
p_2	<0.83,0.1,0.18>	<0.8,0,0.08>	<0.93,0.2,0.1>	<0.8,0.3,0.23>
p_3	<0.93,0.2,0.08>	<0.7,0.1,0.3>	<0.33,0.2,0.53>	<0.48,0.2,0.38>
p_4	<0.75,0.1,0.2>	<0.85,0.3,0.25>	<0.85,0.3,0.2>	<0.85,0.3,0.45>

Table 40. Table presentation of $(\psi,S)_{\Lambda_{d_4}}$ 5th part.

S / P	s_{17}	s_{18}	s_{19}	s_{20}
p_1	<0.95,0.4,0.15>	<0.88,0.3,0.18>	<0.85,0,0.05>	<0.78,0.6,0.08>
p_2	<0.78,0.5,0.08>	<0.8,0.2,0.23>	<0.85,0.2,0.18>	<0.95,0.4,0.13>
p_3	<0.63,0.3,0.53>	<0.55,0.2,0.6>	<0.88,0.5,0.45>	<0.48,0.5,0.3>
p_4	<0.6,0.5,0.15>	<0.55,0.1,0.35>	<0.75,0.2,0.1>	<0.9,0.4,0.3>

Ultimately, the score table is generated by evaluating the similarity of each row in the Tables 21 to 36 and every row in Tables 17 to 20. This includes finding the highest value for each patient and the diseases associated with these values. The following formula is then used to calculate the similarity:

$$T_S = \frac{\sum_l^{20}\left|T_{\psi_{\Lambda_{d_i}}(p_i)(s_l)} - T_{\phi(p_i)(s_l)}\right|}{\sum_l^{20}\left|T_{\psi_{\Lambda_{d_i}}(p_i)(s_l)} + T_{\phi(p_i)(s_l)}\right|},\ I_S = \frac{\sum_l^{20}\left|I_{\psi_{\Lambda_{d_i}}(p_i)(s_l)} - I_{\phi(p_i)(s_l)}\right|}{\sum_l^{20}\left|I_{\psi_{\Lambda_{d_i}}(p_i)(s_l)} + I_{\phi(p_i)(s_l)}\right|} \text{ and}$$

$$F_S = \frac{\sum_l^{20}\left|F_{\psi_{\Lambda_{d_i}}(p_i)(s_l)} - F_{\phi(p_i)(s_l)}\right|}{\sum_l^{20}\left|F_{\psi_{\Lambda_{d_i}}(p_i)(s_l)} + F_{\phi(p_i)(s_l)}\right|}$$

Then, $S(p_i,d_j) = \frac{1-T_S+1-I_S+1-F_S}{3}$

The result can be obtained as follows:

$$1 - T_S = 1 - \frac{|0.93-1|+|0.83-1|+|0.83-1|+|0.8-1|}{|0.93+1|+|0.83+1|+|0.83+1|+|0.8+1|}$$
$$\frac{+|0.95-0.5|+|0.98-0|+|0.83-0.5|+|0.95-0.5|}{+|0.95+0.5|+|0.98+0|+|0.83+0.5|+|0.95+0.5|}$$
$$\frac{+|0.93-0.5|+|0.88-1|+|0.9-0|+|0.95-0.5|}{+|0.93+0.5|+|0.88+1|+|0.9+0|+|0.95+0.5|}$$
$$\frac{+|0.93-0|+|0.83-0|+|0.78-0|+|0.8-0|}{+|0.93+0|+|0.83+0|+|0.78+0|+|0.8+0|}$$
$$\frac{+|0.95-0|+|0.88-0|+|0.85-0|+|0.78-0|}{+|0.95+0|+|0.88+0|+|0.85+0|+|0.78+0|} = 1 - \frac{11.52}{25.06} = 0.54$$

$$1 - I_S = 1 - \frac{|0.2-1|+|0.1-1|+|0.1-1|+|0.3-1|}{|0.2+1|+|0.1+1|+|0.1+1|+|0.3+1|}$$
$$\frac{+|0.3-0.5|+|0.1-0|+|0-0.5|+|0.2-0.5|}{+|0.3+0.5|+|0.1+0|+|0+0.5|+|0.2+0.5|}$$
$$\frac{+|0.1-0.5|+|0.1-1|+|0.2-0|+|0.3-0.5|}{+|0.1+0.5|+|0.1+1|+|0.2+0|+|0.3+0.5|}$$
$$\frac{+|0.1-0|+|0.2-0|+|0.3-0|+|0.1-0|}{+|0.1+0|+|0.2+0|+|0.3+0|+|0.1+0|}$$
$$\frac{+|0.4-0|+|0.3-0|+|0-0|+|0.6-0|}{+|0.4+0|+|0.3+0|+|0+0|+|0.6+0|}$$
$$= 1 - \frac{8.1}{11.5} = 0.29$$

$$1 - F_S = 1 - \frac{|0.13-0|+|0.05-0|+|0.13-0|+|0.13-0|}{|0.13+0|+|0.05+0|+|0.13+0|+|0.13+0|}$$
$$\frac{+|0.15-0.5|+|0.13-1|+|0.2-0.5|+|0.15-0.5|}{+|0.15+0.5|+|0.13+1|+|0.2+0.5|+|0.15+0.5|}$$
$$\frac{+|0.13-0.5|+|0.2-0|+|0.08-1|+|0.1-0.5|}{+|0.13+0.5|+|0.2+0|+|0.08+1|+|0.1+0.5|}$$
$$\frac{+|0.1-1|+|0.23-1|+|0.13-1|+|0.1-1|}{+|0.1+1|+|0.23+1|+|0.13+1|+|0.1+1|}$$
$$\frac{+|0.15-1|+|0.18-1|+|0.05-1|+|0.08-1|}{+|0.15+1|+|0.18+1|+|0.05+1|+|0.08+1|}$$
$$= 1 - \frac{11.18}{15.72} = 0.29$$

$$(p_1,d_1) = \frac{0.54+0.29+0.29}{3} = 0.37$$

Table 41. $S(p_i,d_j)$.

D / p	d_1	d_2	d_3	d_4
p_1	0.37	**0.45**	0.43	0.40
p_2	**0.47**	0.44	0.42	0.43
p_3	0.47	0.33	0.29	**0.53**
p_4	0.53	**0.54**	0.47	0.46

By employing similar calculations, we derive the score table, as shown in Table 41. Analysis of Table 41 reveals that the first sick individual is diagnosed with dengue fever, the second sick individual is suffering from COVID-19, the third sick individual is affected by typhoid, and the fourth sick individual is diagnosed with dengue fever.

8. Conclusions

In this paper, we have introduced the concept of ENSS which is more effective and useful with some of its properties. Also, the basic operations on ENSS namely complement, union and intersection have been defined. Finally, we have presented an application of ENSS in a DM problem and in MD.

This article makes a significant contribution by introducing new inquiries that can inspire further research efforts. As research progresses and reveals additional questions, the content presented in this article establishes a foundation for more thorough investigation into ENSS. Key areas for future research encompass advanced concepts like Q-Effective Neutrosophic Soft Sets (Q-ENSSs) and their applications, effective neutrosophic hypersoft sets with applications, and their generalizations. Additionally, there is a need to examine algebraic structures linked to

ENSS, including groups, rings, fields, or vector spaces, alongside exploring applications in real-world domains.
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Conflicts of Interest

"The authors declare no conflict of interest."

References

[1] Abu Qamar M. and Hassan N., "Generalized Q-Neutrosophic Soft Expert Set for Decision under Uncertainty," *Symmetry*, vol. 10, no. 11, pp. 1-16, 2018. https://www.mdpi.com/2073-8994/10/11/621

[2] Al-Hijjawi S. and Alkhazaleh S., "Possibility Neutrosophic Hypersoft Set," *Neutrosophic Sets and Systems*, vol. 53, no. 1, pp. 117-129, 2023. https://digitalrepository.unm.edu/nss_journal/vol53/iss1/7

[3] Al-Hijjawi S., Ahmad A., and Alkhazaleh S., "Time Q-Neutrosophic Soft Expert Set," *International Journal of Neutrosophic Science*, vol. 19, no. 1, pp. 088-28, 2022. DOI:10.54216/IJNS.190101

[4] Alkhazaleh S., "Effective Fuzzy Soft Set Theory and its Applications," *Applied Computational Intelligence and Soft Computing*, vol. 2022, pp. 1-12, 2022. https://doi.org/10.1155/2022/6469745

[5] Alkhazaleh S. and Beshtawi E., "Effective Fuzzy Soft Expert Set Theory and its Applications," *International Journal of Fuzzy Logic and Intelligent Systems*, vol. 23, no. 2, pp. 192-204, 2023. https://doi.org/10.5391/IJFIS.2023.23.2.192

[6] Alkhazaleh S. and Hazaymeh A., "N-Valued Refined Neutrosophic Soft Sets and their Applications in Decision Making Problems and Medical Diagnosis," *Journal of Artificial Intelligence and Soft Computing Research*, vol. 8, no. 1, pp. 79-86, 2018. DOI:10.1515/jaiscr-2018-0005

[7] Alkhazaleh S. and Salleh A., "Fuzzy Soft Expert Set and its Application," *Applied Mathematics*, vol. 5, no. 9, pp. 1349-1368, 2014. https://doi.org/10.4236/am.2014.59127

[8] Alkhazaleh S. and Salleh A., "Soft Expert Sets," *Advances in Decision Sciences*, vol. 2011, pp. 1-12, 2011. https://doi.org/10.1155/2011/757868

[9] Alkhazaleh S., Salleh A., and Hassan N., "Possibility Fuzzy Soft Set," *Advances in Decision Sciences*, vol. 2011, pp. 1-18, 2011. http://doi.org/10.1155/2011/479756

[10] Alkhazaleh S., Salleh A., and Hassan N., "Soft Multisets Theory," *Applied Mathematical Sciences*, vol. 5, no. 72, pp. 3561-3573, 2011. https://www.researchgate.net/publication/235631549_Soft_multisets_theory

[11] Al-Sharqi F., Al-Qudah Y., and Alotaibi N., "Decision-Making Techniques Based on Similarity Measures of Possibility Neutrosophic Soft Expert Sets," *Neutrosophic Sets and Systems*, vol. 55, no. 1, pp. 358-382, 2023. https://digitalrepository.unm.edu/nss_journal/vol55/iss1/22

[12] Al-Sharqi F., Al-Quran A., and Romdhini M., "Decision-Making Techniques Based on Similarity Measures of Possibility Interval Fuzzy Soft Environment," *Iraqi Journal for Computer Science and Mathematics*, vol. 4, no. 4, pp. 18-29, 2023. https://doi.org/10.52866/ijcsm.2023.04.04.003

[13] Broumi S. and Smarandache F., "Intuitionistic Neutrosophic Soft Set," *Journal of Information and Computing Science*, vol. 8, no. 2, pp. 130-140, 2013. https://www.researchgate.net/publication/333559318_Intuitionistic_Neutrosophic_Soft_Set

[14] Deli I. and Broumi S., "Neutrosophic Soft Matrices and NSM-Decision Making," *Journal of Intelligent and Fuzzy Systems*, vol. 28, no. 5, pp. 2233-2241, 2015. https://doi.org/10.3233/IFS-141505

[15] Hassan N., Uluçay V., and Şahin M., "Q-Neutrosophic Soft Expert Set and its Application in Decision Making," *International Journal of Fuzzy System Applications*, vol. 7, no. 4, pp. 37-61, 2018. https://doi.org/10.4018/IJFSA.2018100103

[16] Maji P., "Neutrosophic Soft Set," *Annals of Fuzzy Mathematics and Informatics*, vol. 5, no. 1, pp. 157-168, 2013. https://fs.unm.edu/Maji-NeutrosophicSoftSet.pdf

[17] Maji P., Biswas R., and Roy A., "Soft Set Theory," *Computers Mathematical with Applications*, vol. 45, no. 4, pp. 555-562, 2003. https://doi.org/10.1016/S0898-1221(03)00016-6

[18] Maji P., Roy A., and Biswas R., "An Application of Soft Sets in a Decision Making Problem," *Computers Mathematical with Applications*, vol. 44, no. 8, pp. 1077-1083, 2002. https://doi.org/10.1016/S0898-1221(02)00216-X

[19] Majumdar P. and Samanta S., "Generalized Fuzzy Soft Sets," *Computers and Mathematics with Applications*, vol. 59, no. 4, pp. 1425-1432, 2010. https://doi.org/10.1016/j.camwa.2009.12.006

[20] Molodtsov D., "Soft Set Theory-first Results," *Computers and Mathematics with Applications*,

vol. 37, no. 2, pp. 19-31, 1999. https://doi.org/10.1016/S0898-1221(99)00056-5

[21] Roy A. and Maji P., "A Fuzzy Soft Set Theoretic Approach to Decision Making Problems," *Journal of Computational and Applied Mathematics*, vol. 203, no. 2, pp. 412-418, 2007. https://doi.org/10.1016/j.cam.2006.04.008

[22] Saeed M., Ahsan M., Siddique M., and Ahmad M., "A Study of the Fundamentals of Hypersoft Set Theory," *International Journal of Scientific and Engineering Research*, vol. 11, no. 1, pp. 220-239, 2020. https://www.ijser.org/researchpaper/A-Study-of-The-Fundamentals-of-Hypersoft-Set-Theory.pdf

[23] Sahin M., Alkhazaleh S., and Ulucay V., "Neutrosophic Soft Expert Sets," *Applied Mathematics*, vol. 6, no. 1, pp. 116-127, 2015. https://doi.org/10.4236/am.2015.61012

[24] Saqlain M., Moin S., Jafar M., Saeed M., and Smarandache F., "Aggregate Operators of Neutrosophic Hypersoft Set," *Neutrosophic Sets and Systems*, vol. 32, no. 1, pp. 294-306, 2020. https://digitalrepository.unm.edu/nss_journal/vol32/iss1/18

[25] Saqlain M., Jafar N., Moin S., Saeed M., and Broumi S., "Single and Multi-Valued Neutrosophic Hypersoft Set and Tangent Similarity Measure of Single Valued Neutrosophic Hypersoft Sets," *Neutrosophic Sets and Systems*, vol. 32, no. 1, pp. 317-329, 2020. https://api.semanticscholar.org/CorpusID:216486181

[26] Saqlain M. and Xin X., "Interval Valued, m-Polar and m-Polar Interval Valued Neutrosophic Hypersoft Sets," *Neutrosophic Sets and Systems*, vol. 36, pp. 389-399, 2020. https://api.semanticscholar.org/CorpusID:226439299

[27] Smarandache F., *A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics*, InfoLearnQuest, 2006. https://zenodo.org/records/49174 (fifth edition)

[28] Smarandache F., "Extension of Soft Set to Hypersoft Set, and then to Plithogenic Hypersoft Set," *Neutrosophic Sets and Systems*, vol. 22, no. 1, pp. 168-170, 2018. file:///C:/Users/user/Downloads/ExtensionOfSoftSetToHypersoft.pdf

[29] Uluay V., Ahin M., and Hassan N., "Generalized Neutrosophic Soft Expert Set for Multiple-Criteria Decision-Making," *Symmetry*, vol. 10, no. 10, pp. 1-17, 2018. https://doi.org/10.3390/sym10100437

[30] Yan L., "Modeling Fuzzy Data with Fuzzy Data Types in Fuzzy Database and XML Models," *The International Arab Journal of Information Technology*, vol. 10, no. 6, pp. 43-48, 2013. shttps://www.ccis2k.org/iajit/PDF/vol.10,no.6/4847.pdf

[31] Zadeh L., "Fuzzy Sets," *Information and Control*," vol. 8, no. 3, pp. 338-253, 1965. https://doi.org/10.1016/S0019-9958(65)90241-X

[32] Zulqarnain R., Xin X., Saqlain M., and Smarandache F., "Generalized Aggregate Operators on Neutrosophic Hypersoft Set," *Neutrosophic Sets and Systems*, vol. 36, no. 1, pp. 271-281, 2020. https://digitalrepository.unm.edu/nss_journal/vol36/iss1/20

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